

7.4 Composing and Decomposing

A Develop Understanding Task

As the day get warmer, you and your friends decide to cool off by taking a ride on the *Turbulent Waters Dive*. As you are waiting in line your tour guide explains the mathematics behind designing the waiting area for a ride.

“As you can see,” says the engineer, “the waiting area can be enlarged or reduced by moving a few chains around. The area we need for waiting guests depends on the time of day. We collect data for each ride so we can use functions to model the typical wait time and how much waiting area we need to provide for our guests.”

And of course, your guide has the functions that represent this particular ride.



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- **Average number of people in the TWD line as a function of time:** $p(t) = 3000 \cos\left(\frac{1}{5}(t - 3)\right)$

→ t is the number of hours before or after noon, so $t = 2$ represents 2:00 p.m. and $t = -2$ represents 10:00 a.m.

→ p represents the number of people in line

- **Waiting area required as a function of the number of people in line:** $a(p) = 4p + 100$

→ a , the waiting area, is measured in square feet

- **Wait time for a guest as a function of the number of people in line:** $W(p) = 60 \cdot \left(\frac{p - 1500}{1500}\right)$

→ W , the wait time, is measured in minutes

1. How much waiting area is required for the guests in line for the *Turbulent Waters Dive* at each of the times listed in the following table?

<i>Time of Day</i>	<i>Waiting Area Required (sq. ft.)</i>
10:00 a.m.	
12:00 noon	
2:00 p.m.	
4:00 p.m.	
8:00 p.m.	



- a. For each instant in time you had to complete a series of calculations. Describe how you found the waiting area at different times.
- b. Can you create a single rule that will determine the waiting area as a function of the time of day?
2. What is the wait time for a guest that arrives at the end of the line for the *Turbulent Waters Dive* at each of the times listed in the following table?

<i>Time of Day</i>	<i>Wait Time (minutes)</i>
10:00 a.m.	
12:00 noon	
2:00 p.m.	
4:00 p.m.	
8:00 p.m.	

- a. For each instant in time you had to complete a series of calculations. Describe how you found the wait time at different times of the day.
- b. Can you create a single rule that will determine the wait time as a function of the time of day?

To maintain crowd control when the lines get long, cast members dressed as pirates (the *Turbulent Waters Dive* has a pirate theme) mingle with the waiting guests. Their antics distract the guests who listen attentively to their pirate jokes. The number of cast members needed depends on the number of people waiting in the line.

• **Number of ushers needed as a function of the number of people in line:**

$$c(t) = \frac{p}{150}$$

→ p represents the number of people in line

→ c represents the number of cast members needed



3. How many cast members are needed to entertain and distract the waiting guests at each of the following times of the day?

<i>Time of Day</i>	<i>Cast Members Needed</i>
10:00 a.m.	
12:00 noon	
2:00 p.m.	
4:00 p.m.	
8:00 p.m.	
t hours before or after noon ($t < 0$ before noon, $t > 0$ after noon)	

On warm, sunny days misters are used to cool down the waiting guests. The number of misters that need to be turned on depends on the size of the waiting area that has been opened up to contain the number of people in line.

- **Number of misters needed as a function of the waiting area:**

$$m(t) = \frac{a}{1000}$$

→ a , the waiting area, is measured in square feet

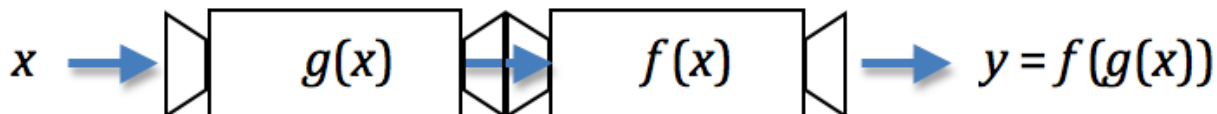
→ m represents the number of misters to be turned on

4. How many misters need to be turned on to cool the waiting guests at each of the following times of day?

<i>Time of Day</i>	<i>Misters Needed</i>
10:00 a.m.	
12:00 noon	
2:00 p.m.	
4:00 p.m.	
8:00 p.m.	
t hours before or after noon ($t < 0$ before noon, $t > 0$ after noon)	



5. Explain how the following diagram might help you think about the work you have been doing on the previous problems. How does the notation used in the diagram support the way you have been combining functions in this task? This way of combining functions is called **function composition**.



Interpreting the Functions

6. At what time of day is the number of people in line the largest?
 What is the maximum number of people in line, based on the function for the average number of people in line?
 When do you think the amusement park opens and closes, based on this function?
7. In terms of the story context, what do you think the 4 and the 100 represent in function rule for waiting area?
8. In terms of the story context, what might be the meaning of the 1500 in the function rule for wait time?
9. In terms of the story context, what might be the meaning of the 150 in the function rule for cast members needed?
10. In terms of the story context, what might be the meaning of the 1000 in the function rule for the number of misters needed?



Name _____

Modeling With Functions | 7.4

Ready, Set, Go!**Ready**

Topic: Recognizing operations on a variable

Each expression contains 2 operations. One of the operations will be “inside” the second operation. Identify the “inside” operation as u by writing $u = \underline{\hspace{2cm}}$. Then substitute u into the expression so that the “outside” operation is being performed on u .

Example: *Given:* $5x^3$.

I can see two operations on x . First the x is being cubed and then x^3 is multiplied by 5. So if $u = x^3$, then $5x^3 = 5u$.

1. Would the answer in the example have been different if you were given $(5x)^3$? Explain

2. $(x - 6)^2$

 $u =$

3. $\tan(x + 4)$

 $u =$

4. $\sqrt[3]{(2x - 7)}$

 $u =$

5. $-9(x + 5)$

 $u =$

6. $\frac{5}{x^2}$

 $u =$

7. $(\sin x)^4$

 $u =$ 

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Modeling With Functions | 7.4

Set Topic: Creating formulas for composite functions Recall that $f(g(x)) = (f \circ g)(x)$.

8. Let $f(x) = 2x^2 - 4$ and $g(x) = 5x$. Find:

a) $(f \circ g)(1)$ b) $(g \circ f)(1)$ c) $(f \circ f)(-2)$ d) $(g \circ g)(-1)$

9. Let $f(x) = \frac{8}{x-3}$ and $g(x) = \frac{15}{x+1}$.

Find: a) $(f \circ g)(x)$ b) $(g \circ f)(x)$ c) $(f \circ f)(x)$ d) $(g \circ g)(x)$

10. Use the functions in #9. Find: a) $(f \circ g)(2)$ b) $(g \circ f)(-5)$

11. Describe the problem that you encountered when calculating the values in #10.

12. Describe the domains for a) $(f \circ g)(x)$ b) $(g \circ f)(x)$ c) $(f \circ f)(x)$ d) $(g \circ g)(x)$

13. What makes the domain for each composition different?



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Modeling With Functions | 7.4

Go Topic: Writing equations of polynomials given the degree and the roots**Write the equation of the polynomial with the given features.**

	Degree of polynomial	Given roots (you may have to determine others):	Leading coefficient	Equation in standard form:
14.	3	-2, 1, and -1	3	
15.	4	$(2 + i)$, 4, 0	1	
16.	5	1 multiplicity 2, -1 multiplicity 2, and 3	-1	
17.	4	$(3 - i)$, $\sqrt{2}$	-2	

