7.1: Function Family Reunion

Characteristics of Function Families

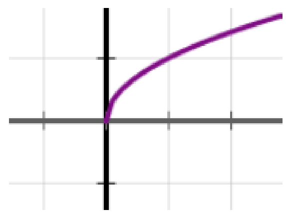
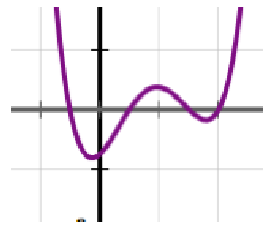
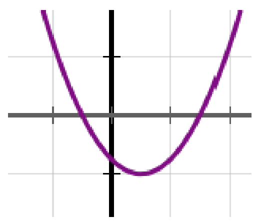
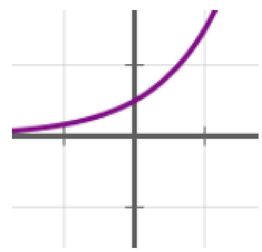
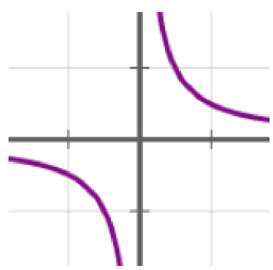
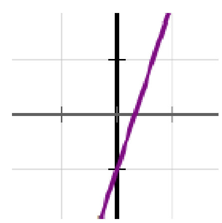
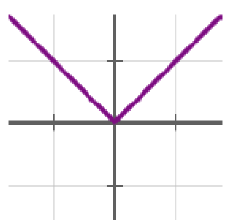
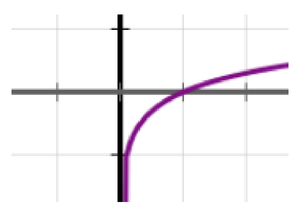
During the past few years of math classes you have studied a variety of functions: linear, exponential, quadratic, polynomial, rational, radical, absolute value, logarithmic and trigonometric. Like a family, each of these types of functions has similar characteristics that differ from other types of functions, making them uniquely qualified to model specific types of real world situations. Because of this, sometimes we refer to each type of function as a "family of functions."

1. Match each function family with the algebraic notation that best defines it.

Function Family Name	Algebraic Description of the Parent Function
1. *linear*	**A.** $y = \lvert x \rvert$
2. *exponential*	**B.** $y = a\sin(bx)$ *or* $y = a\cos(bx)$ *or* $y = a\tan(bx)$
3. *quadratic*	**C.** $y = mx + b$
4. *polynomial*	**D.** $y = \log(x)$
5. *rational*	**E.** $y = ax^2 + bx + c$
6. *absolute value*	**F.** $y = \frac{1}{x}$
7. *logarithmic*	**G.** $y = a \cdot b^x$
8. *trigonometric*	**H.** $y = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_0$
9. *radical*	**I.** $y = \sqrt[n]{x}$

Just like your family, each member of a function family resembles other members of the family, but each has unique differences, such as being “wider” or “skinnier”, “taller” or “shorter”, or other features that allow us to tell them apart. We might say that each family of functions has a particular “genetic code” that gives its graph its characteristic shape. We might refer to the simplest form of a particular family as “the parent function” and consider all transformations of this parent function to be members of the same family.

2. Match each function family with the characteristic shape of the graph that fits it.

Function Family Name	**Characteristic Shape of the Graph**	
1. *linear*	**A.** 	**B.** 
2. *exponential*		
3. *quadratic*	**C.** 	**D.** 
4. *polynomial*		
5. *rational*	**E.** 	**F.** 
6. *absolute value*		
7. *logarithmic*	**G.** 	**H.** 
8. *trigonometric*		
9. *radical*	**I.** 	

Function family characteristics are passed on to their "children" through a variety of transformations. While the members of each family shares common characteristics, transformations make each member of a family uniquely qualified to accomplish the mathematical work they are required to do.

3. For each of the following tables, a set of coordinate points that captures the characteristics of a parent graph is given. The additional columns give coordinate points for additional members of the family after a particular transformation has occurred. Write the rule for each of the different transformations of the parent graph. (Note: We can think of each new set of coordinate points (that is, the *image* points) as a geometric transformation of the original set of coordinate points (that is, the *pre-image* points) and use the notation associated with geometric transformations to describe transformation. Or, we can write the rule using algebraic function notation. Use both types of notation to represent each transformation.)

	pre-image (parent graph)	*image 1*	*image 2*	*image 3*
geometric notation	(x, y)	$(x, y) \rightarrow (x, y+2)$		
function notation	$f(x) = x^2$	$f_1(x) = x^2 + 2$		
selected points that fit this image	(-2, 4)	(-2, 6)	(-2, 8)	(-3, 4)
	(-1, 1)	(-1, 3)	(-1, 2)	(-2, 1)
	(0, 0)	(0, 2)	(0, 0)	(-1, 0)
	(1, 1)	(1, 3)	(1, 2)	(0, 1)
	(2, 4)	(2, 6)	(2, 8)	(1, 4)

	pre-image (parent graph)	*image 1*	*image 2*	*image 3*
geometric notation	(x, y)			
function notation	$f(x) = 2^x$			
selected points that fit this image	(-2, ¼)	(-2, 1)	(-2, -¼)	(-3, ¼)
	(-1, ½)	(-1, 2)	(-1, -½)	(-2, ½)
	(0, 1)	(0, 4)	(0, -1)	(-1, 1)
	(1, 2)	(1, 8)	(1, -2)	(0, 2)
	(2, 4)	(2, 16)	(2, -4)	(1, 4)

	pre-image (parent graph)	*image 1*	*image 2*	*image 3*
geometric notation	(x, y)			
function notation	$f(x) = \lvert x \rvert$			
selected points that fit this image	(-2, 2)	(-2, -4)	(2, 2)	(-5, 2)
	(-1, 1)	(-1, -2)	(3, 1)	(-4, 1)
	(0, 0)	(0, 0)	(4, 0)	(-3, 0)
	(1, 1)	(1, -2)	(5, 1)	(-2, 1)
	(2, 2)	(2, -4)	(6, 2)	(-1, 2)

	pre-image (parent graph)	*image 1*	*image 2*	*image 3*
geometric notation	(x, y)			
function notation	$f(x) = \sin(x)$			
selected points that fit this image	(0, 0)	(0, 2)	(0, 0)	(0, 0)
	$(\frac{\pi}{2}, 1)$	$(\frac{\pi}{2}, 3)$	$(\frac{\pi}{4}, 1)$	$(\frac{\pi}{2}, -2)$
	$(\pi, 0)$	$(\pi, 2)$	$(\frac{\pi}{2}, 0)$	$(\pi, 0)$
	$(\frac{3\pi}{2}, -1)$	$(\frac{3\pi}{2}, 1)$	$(\frac{3\pi}{4}, -1)$	$(\frac{3\pi}{2}, 2)$
	$(2\pi, 0)$	$(2\pi, 2)$	$(\pi, 0)$	$(2\pi, 0)$

	pre-image (parent graph)	*image 1*	*image 2*	*image 3*
geometric notation	(x, y)			
function notation	$f(x) = \sqrt{x}$			
selected points that fit this image	(0, 0)	(0, 0)	(0, 0)	(3, 0)
	(1, 1)	$(1, \frac{1}{2})$	$(\frac{1}{2}, 1)$	(4, 1)
	(4, 2)	(4, 1)	(2, 2)	(7, 2)
	(9, 3)	$(9, \frac{3}{2})$	$(\frac{9}{2}, 3)$	(12, 3)
	(16, 4)	(16, 2)	(8, 4)	(19, 4)