3.2 Component Form Vector Operations

Practice Tasks

I. Concepts and Procedures

1. The vector $\langle a, b \rangle$ has _____ component a and _____ component b.

2. The _____ of the vector u is denoted by $||u||$.

3. Express the vector with initial point P and terminal point Q in component form.
   a. $P(3,2);\ Q(8,9)$
   b. $P(1,1);\ Q(9,9)$
   c. $P(5,3);\ Q(1,0)$
   d. $P(-1,3);\ Q(-6,-1)$
   e. $P(-1,-1);\ Q(-1,1)$

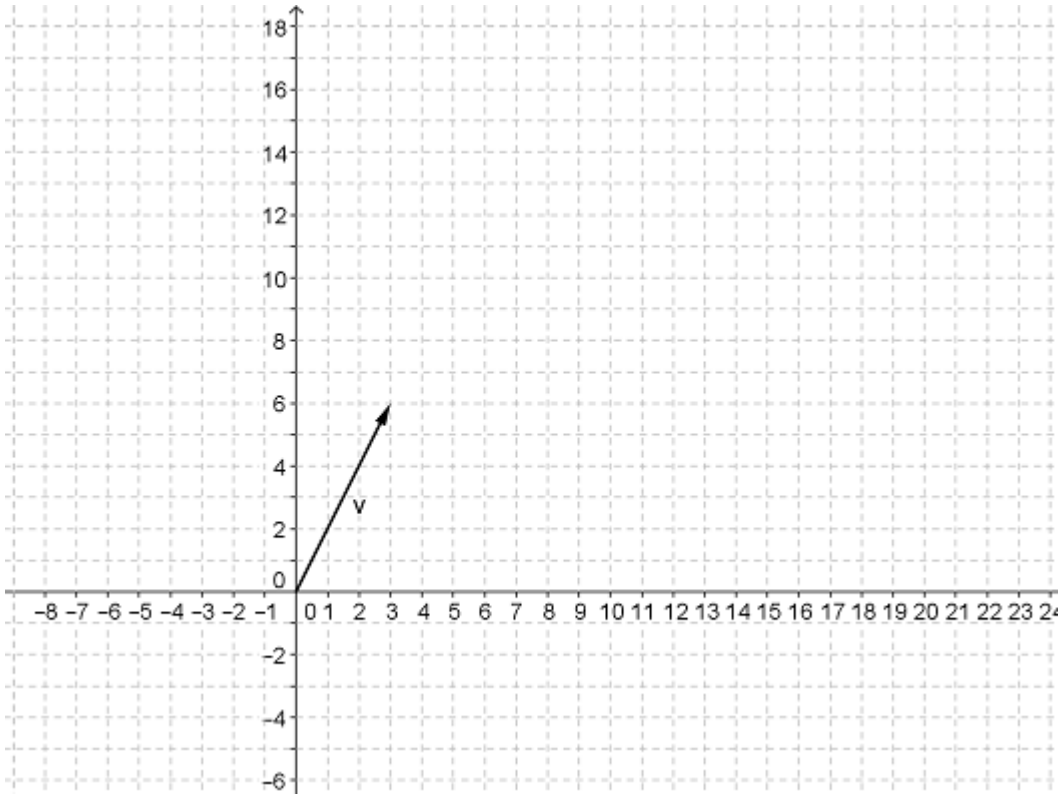
4. Given $\mathbf{u} = \langle \mathbf{3}, \mathbf{1} \rangle$ and $\mathbf{v} = \langle -\mathbf{4}, \mathbf{2} \rangle$, write each vector in component form, graph it, and explain the geometric effect.

   a. $3u$
   b. $\frac{1}{2}v$
   c. $-2u$
   d. $-v$
   e. $u + v$
   f. $2u + 3v$
   g. $4u - 3v$
   h. $\frac{1}{2}u - \frac{1}{3}v$

5. Given the vectors shown below.

$$\mathbf{v} = \langle 3,6 \rangle$$
$$\mathbf{u} = \langle 9,18 \rangle$$
$$\mathbf{w} = \langle -3,-6 \rangle$$
$$\mathbf{s} = \langle 1,2 \rangle$$
$$\mathbf{t} = \langle -1.5,-3 \rangle$$
$$\mathbf{r} = \langle 6,12 \rangle$$

a. Draw each vector with its initial point located at $(0,0)$. The vector $\mathbf{v}$ is already shown. How are all of these vectors related?



b. Which vector is $2\mathbf{v}$? Explain how you know.

c. Describe the remaining vectors as a scalar multiple of $\mathbf{v} = \langle 3,6 \rangle$ and explain your reasoning.

d. Is the vector $\mathbf{p} = \langle 3\sqrt{2}, 6\sqrt{2} \rangle$ a scalar multiple of $\mathbf{v}$? Explain.

e. Which vector would it make sense to call the opposite of $\mathbf{v} = \langle 3,6 \rangle$?

6. Given $\mathbf{u} = \langle 3,1 \rangle$ and $\mathbf{v} = \langle -4,2 \rangle$, find the following.

a. $\|\mathbf{u}\|$

b. $\|\mathbf{v}\|$

c. $\|2\mathbf{u}\|$ and $2\|\mathbf{u}\|$

d. $\left\|\frac{1}{2}\mathbf{v}\right\|$ and $\frac{1}{2}\|\mathbf{v}\|$

e. Is $\|\mathbf{u} + \mathbf{u}\|$ equal to $\|\mathbf{u}\| + \|\mathbf{u}\|$? Explain how you know.

f. Is $\|\mathbf{u} + \mathbf{v}\|$ equal to $\|\mathbf{u}\| + \|\mathbf{v}\|$? Explain how you know.

g. Is $\|\mathbf{u} - \mathbf{v}\|$ equal to $\|\mathbf{u}\| - \|\mathbf{v}\|$? Explain how you know.

II. Problem Solving

1. To commute to school, Larisa leaves her house and drives south on Maple Ave. for 2.4 miles. She turns East on Cottage Grove Rd for 3.1 miles, and then turns South on Blue Hills Ave. for 2.9 miles Finally she turns East on Tower Ave. and after driving 1.3 miles, she arrives at her school. Express Larissa's commute (from home to school) as a vector in component form.

2. Sergei experienced one of the biggest earthquakes when visiting Taiwan in 1999. He noticed that his refrigerator moved on the wooden floor and made marks on it. By measuring the marks he was able to trace how the refrigerator moved. The first move was northeast with a distance of 20 cm. The second move was northwest with a distance of 10 cm. The final move was northeast with a distance of 5 cm. Find the vectors that would re-create the refrigerator's movement on the floor and find the distance that the refrigerator moved from its original spot to its resting place. Draw a diagram of these vectors.

III. Reasoning

1. Why is the vector $\mathbf{o} = \langle 0,0 \rangle$ called the zero vector? Describe its geometric effect when added to another vector.