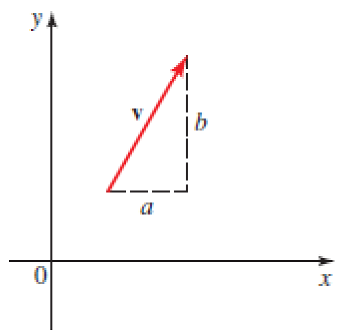
3.2: Shopping in Hogsmeade

Vector Operations in Component Form

Writing the component form of a vector looks a lot like a coordinate pair, but uses the symbols "⟨" and "⟩" instead of "(" and ")". We can easily represent Harry's paths using this notation: $v = \langle a, b \rangle$, where a is the **horizontal component** and b is the **vertical component** of vector v.



We can also represent the **magnitude** of a vector *v* using double bars: $||v||$. (Some people use single bars instead.)

1. Complete the table below to practice this notation—remember positive and negative signs!

Tuesday – Start at Hogwarts and travel:				
	East/West	**North/South**	**Component Form**	**Magnitude**
Stop 1:	2 blocks East	3 blocks North	$s = \langle 2,3 \rangle$	$\|\|s\|\| = \sqrt{13}$
Stop 2:	4 blocks West	2 blocks North	$t = \langle -4,2 \rangle$	$\|\|t\|\| =$
Stop 3:	3 blocks East	1 block South	$u =$	$\|\|u\|\| =$
Stop 4:	4 blocks West	3 blocks South	$v =$	$\|\|v\|\| =$
Hogwarts:	3 blocks East	1 block South	$w =$	$\|\|w\|\| =$

2. a. At Stop 2, how far East or West is Harry from Hogwarts?

   b. How far North or South?

II. Hopping from Shop to Shop in Hogsmeade

In Lesson 1, we looked at the difference between walking and flying to Hogsmeade from Hogwarts. Now that you know vector terminology, see if you can help Harry and his friends on a shopping trip in Hogsmeade. The following table gives the coordinates of several shops in Hogsmeade:

Location	Coordinates
Dervish & Bang	(-3,4)
Gladrags	(2,3)
Hogsmeade Station	(0,0)
Madam Puddifoots	(3,-2)
Potions DeSilva	(-2,-1)
Scrivenshafts	(9,4)

3. Write the component-form vector for the following shopping hops:
   a. From Dervish & Bang to Gladrags:

   b. From Madam Puddifoots to Potions DeSilva:

   c. From Scrivenshafts to Potions DeSilva:

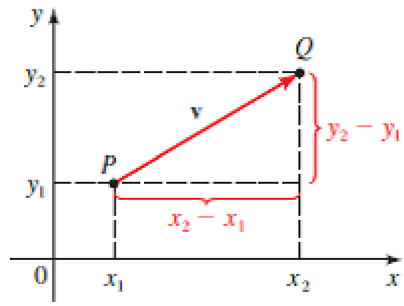
   d. From Potions DeSilva to Scrivenshafts:

   e. From Gladrags to Dervish & Bang:

   f. From Hogsmeade Station to Madam Puddifoots:

4. Describe the method you used to determine the vectors above.

Septima Vector (Professor of Arithmancy) says, "To determine the component form of a vector, given its initial and terminal points, just subtract the corresponding coordinates."



COMPONENT FORM OF A VECTOR

If vector v has initial point (x_1,y_1) and terminal point (x_2,y_2), then $v = \langle x_2 - x_1, y_2 - y_1 \rangle$

EXAMPLE 1 – Describing Vectors in Component Form

a. Find the component form of the vector u with initial point (5, -2) and terminal point (7,3).

SOLUTION

a. The desired vector is:

$$u = \langle 7 - 5, 3 - (-2) \rangle = \langle 2,5 \rangle$$

<u>Your Turn</u>:

5. Find the component form of the vectors with:

   a. initial point (3, 6) and terminal point (11,3).

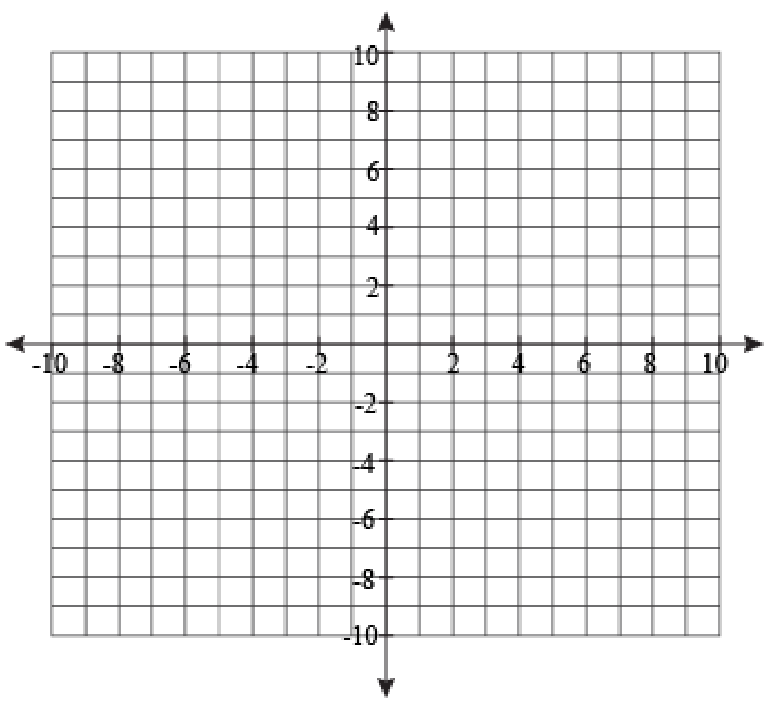
   b. initial point (-5,8) and terminal point (1,4).

   c. initial point (-7, -6) and terminal point (-4,3).

6. Ron complains "I never remember which one to subtract from the other" and Hermione replies "It really doesn't matter… as long as you subtract both the x- and y-coordinates in the same way." Use the following problem to test Hermione's conjecture.
   a. If the vector $v = \langle 3,6 \rangle$ is sketched with initial point (-2, 1), what is its terminal point?

7. If the vector $v = \langle -4,5 \rangle$ is sketched with *terminal* point (2, -4), what is its *initial* point?

8. On the grid below, sketch representations of the vector $w = \langle 2.3 \rangle$ with initial points at (0,0), (2,2), (-2,-1) and (1,4).
   a. What type of vectors is formed? Why?



III. Vector Operations

9. Add the following vectors component-wise (see Professor Vector's explanation on page 3, if you don't remember.)

    a. $\langle 3,5 \rangle + \langle 4,6 \rangle =$

    b. $\langle -3,4 \rangle + \langle -4,2 \rangle =$

    c. $\langle 7,-5 \rangle + \langle 9,-1 \rangle =$

    d. $\langle -2,0 \rangle + \langle 3,-8 \rangle =$

We can also easily perform **vector subtraction** in a similar way.

10. Ron and Hermione both begin at Hogwarts. Ron walks four blocks East and six blocks North to arrive at Zonko's Joke Shop. Hermione walks five blocks East and three blocks South to arrive at the Shrieking Shack. Give Ron directions to get to the Shrieking Shack from Zonko's Joke Shop.

11. Write a vector subtraction problem to represent the steps you took to answer #8.

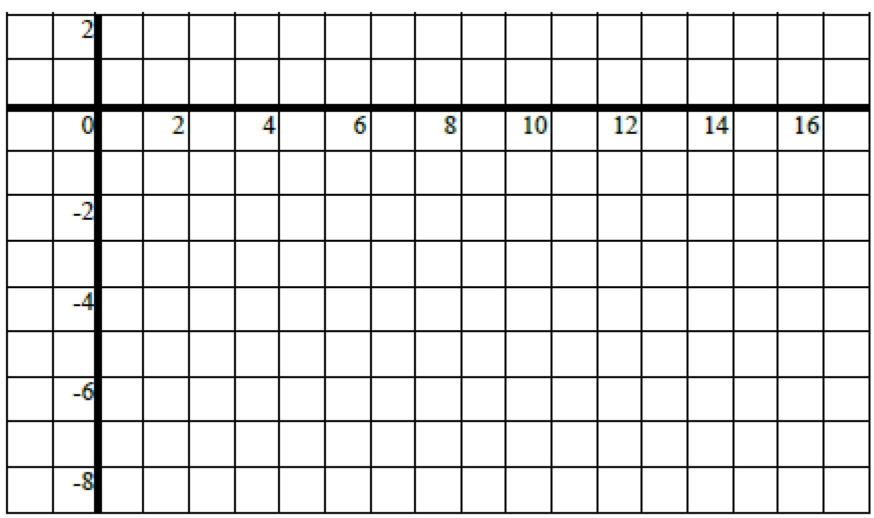
In addition to adding and subtracting vectors, we can also multiplying a vector by a real number, which we call a **scalar.**

12. Use component-wise addition to find ⟨4, -2⟩ + ⟨4, -2⟩ + ⟨4, -2⟩.

We can rewrite the sum ⟨4, -2⟩ + ⟨4, -2⟩ + ⟨4, -2⟩ as the product 3⟨4, -2⟩. This is similar to writing 7 + 7 + 7 as 3(7).

13. Describe a simpler way to find 3⟨4, -2⟩.

14. Let's see what this means graphically. Use the tail-to-end method to represent the sum ⟨4, -2⟩ + ⟨4, -2⟩ + ⟨4, -2⟩ on the grid below.



15. The vector *u* is drawn below. For each problem, draw the new vector.

a. $3u$ b. $\frac{1}{2}u$ c. $-2u$ d. $-u$

16. What types of scalars result in a vector that:
    e. is longer than the original vector?

    f. is shorter than the original vector?

    g. points in the opposite direction of the original vector?

ALGEBRAIC OPERATIONS ON VECTORS

If $u = \langle a_1, b_1 \rangle$ and $v = \langle a_2, b_2 \rangle$, then:

- $u + v = \langle a_1 + a_2, b_1 + b_2 \rangle$
- $u - v = \langle a_1 - a_2, b_1 - b_2 \rangle$
- $c \cdot u = \langle ca_1, cb_1 \rangle$ for any Real number c

17. If $u = \langle 2, -3 \rangle$ and $v = \langle -1, 2 \rangle$, find:
    a. $u + v$

    b. u - v

    c. $2u$

    d. -3v

    e. $2u + 3v$

IV. Flying with Hedwig and Errol

Harry and Ron are both at Zonko's Joke Shop and call to their owls. Hedwig flies directly there, but Ron's clumsy owl? Well, you have to see it to believe it!

18. Open GeoGebra and use the **Point Tool** to locate Zonko's at (4,6).
    a. Use the **Vector Tool** (3rd button - dropdown menu) to draw Errol's indirect route:
        i. Select, in order (0,0) and (1,5)
        ii. Select, in order (1,5) and (4,6)

    b. Record the <u>matrix form</u> of vector *u* below, as seen in the Algebra View window.

    c. Rewrite vector *v* in component form below:

    d. Describe what the numbers 3 and 1 refer to, in terms of the context of the problem.

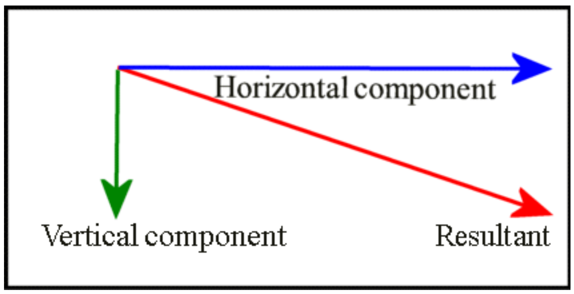
    e. Use the **Vector Tool** to draw Hedwig's direct route from (0,0) to (4,6).

    f. Explain how you know that $u + v = w$.

    g. Rewrite $u + v = w$ using the component form of each of the three vectors.

h. Write a clear set of instructions on adding vectors in component form. (You might want to recall how Septima Vector described how to describe component form on the top of page 3.)

When we add two or more vectors this results in what is called the **resultant vector.**



$v = \langle a,b \rangle$