## 3.1: Walking and Flying Around Hogsmeade

## Introduction to Vectors



Harry Potter needs to make a few stops around Hogsmeade. Harry's broom is broken, so he must walk between the buildings.

The town is laid out in square blocks, which makes it easy to give directions. Here are the directions Harry must follow on Monday:

| Monday - Start at Hogwarts and travel: |  |  |
| :--- | :---: | :---: |
|  | East/West | North/South |
| Stop 1: | 3 blocks East | 5 blocks North |
| Stop 2: | 5 blocks East | 2 blocks North |
| Stop 3: | 2 blocks East | 1 block North |
| Return to Hogwarts: |  |  |
| TOTAL | $\ldots$ | blocks East |
|  | $\ldots$ | blocks North |
|  |  | blocks West |
|  |  |  |

1. Use a piece of graph paper and draw Harry's trip. Put Hogwarts at the origin.
2. Fill in the blank parts of the table to give Harry directions to get back to Hogwarts. (Make your directions simple so that Harry must make only one turn.)
3. How many total blocks East did Harry walk? How many blocks West? Record this in your table. What do you notice?
4. How many total blocks North did Harry walk? How many blocks South? Record this in your table. What do you notice?

II. Walking Around Hogsmeade

On Tuesday, Harry has more errands to run. Here are his new directions:

| Tuesday - Start at Hogwarts and travel: |  |  |
| :---: | :---: | :---: |
|  | East/West | North/South |
| Stop 1: | 2 blocks East | 3 blocks North |
| Stop 2: | 4 blocks West | 2 blocks North |
| Stop 3: | 3 blocks East | 1 block South |
| Stop 4: |  |  |
| Return to Hogwarts: | 3 blocks East | 1 block South |
| TOTAL | $\qquad$ blocks East $\qquad$ blocks West | $\qquad$ blocks North $\qquad$ blocks South |

5. Fill in the blank parts of the table to ensure that Harry's directions will get him back to Hogwarts.
6. Find the total distances East, West, North, and South Harry traveled, and record these in your table. What do you notice?

III. Flying Around Hogsmeade

Harry's trusted owl, Hedwig, can fly over buildings, so she travels in a straight line from each stop to the next and waits for Harry to arrive.
7. On your graph paper from \#1, use a different color to draw arrows representing Hedwig's path.

In mathematics, we use directed line segments, or vectors, to indicate a magnitude (length or distance) and a direction. Each part of Hedwig's trip has a distance and a direction, so the arrows you just drew are vectors.
8. To get from Hogwarts to Stop 1 on Monday, how far did Hedwig fly? (Hint: Use Harry's path on your graph paper as legs of a right triangle.)
9. There are several ways to describe Hedwig's direction during this leg of the trip. We could simply say Hedwig traveled "northeast," but this would not be a very accurate description. Why not?
10. For more accuracy, we can include an angle relating a direction to the nearest cardinal direction (N, S, E, W). Fill in the blanks below to describe each of these directions.

a. $30^{\circ} \mathrm{N}$ of E
b. $20^{\circ}$ $\qquad$ of N
c. $30^{\circ}$ $\qquad$ of $\qquad$ d. $\qquad$
11. This notation can be cumbersome, so mathematicians measure directions as angles (possibly greater than $180^{\circ}$ ) measured counterclockwise from due East called "standard position angles". Rewrite the angles above using this notation. Two of the four have been done for you.
a. $30^{\circ}$
b. $\qquad$ c. $210^{\circ}$
d. $\qquad$
12. Using inverse trigonometry, find Hedwig's direction going from Hogwarts to Stop 1 on Monday. Express your answer in the simple form introduced in \#11. (Hint: Use Harry's path on your graph paper as legs of a right triangle.)
13. Find the magnitude (distance) and direction of Hedwig's path from Stop 1 to Stop 2 on Monday. Show your work neatly.
14. Find the magnitude and direction of Hedwig's path from Stop 2 to Stop 3 on Monday. Show your work neatly.
15. Find the magnitude and direction of Hedwig's path from Stop 3 to Hogwarts on Monday. Show your work neatly. BE CAREFUL! The angle of the triangle is not the same as the angle the path makes with due East. Look at \#11 to see how to make the necessary adjustments.

The way we have expressed Harry's path is known as component form, since it is split up into two parts, or components-a horizontal part and a vertical part. The way we have expressed Hedwig's path is known as magnitude-direction form, since it gives the magnitude and direction of the path.It is important to be able to convert from one form to another.
16. Practice this skill by filling in the table below using the Pythagorean Theorem, trigonometry, and inverse trigonometry. It will probably be helpful to draw a picture of Harry's path (with horizontal and vertical components) and Hedwig's path (a straight flight) to create a right triangle.

|  | Harry's Description |  | Hedwig's Description |  | Drawing |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | horizontal | vertical | magnitude | direction |  |
| a. | 3 blocks <br> East | 4 blocks North |  |  |  |
| b. |  |  | $\begin{gathered} 13 \\ \text { blocks } \end{gathered}$ | $113^{\circ}$ |  |
| c.. | 6 blocks <br> West | 2 blocks South |  |  |  |
| d. |  |  | 10 <br> Blocks | $315^{\circ}$ |  |

## IV. Harry in GeoGebra Land

We will now check our results from the previous exercise with GeoGebra.

- Open a blank GeoGebra from https://www.geogebra.org/
- CW 16a: 3 Blocks East; 4 blocks North
o Turn the Grid On
- Expand the Toggle Bar in the upper right hand corner

o Select the Segment Tool (all the buttons have drop down values - see at right)
o Draw a segment from the origin 3 units east (right)
- Make sure you end precisely at $B(3,0)$
o Draw a segment from $A(3,0) 4$ blocks North (up) to B

o Draw a segment from the origin to $C(3,4)$ to complete the triangle
o Measure $\angle B A C$.
- Select Measure Angle Tool (7th button)
- Select (in order): point B, point A, and point C (make sure you are measuring the acute angle)
o Measure the magnitude (length) of the vector
- Select Distance or Length Tool (same as Measure Angle button)
o Check values against your solution from 15 a
- CW 16b: Magnitude = 13; Direction $=113^{\circ}$
o Delete all your previous work (or open a new graph)
o Select the Segment Tool
o Draw a segment from the origin 13 units east
- Make sure you end precisely at $A(13,0)$

0 Select the Rotate Around Point Tool (9th button)

- Select (in order) segment AB, point A and enter 113 in dialog box
o Get coordinates of B' and interpret as North/South and East/West
- CW 16c - Follow step for 16a, above
- CW 16d - Follow step for 16b, above

17. Open a new GeoGebra graph, Select the Vector Tool (2 $2^{\text {nd }}$ button) and draw three vectors: from $(0,0)$ to $(5,6)$; from $(0,2)$ to $(5,8)$; and from $(0,-2)$ to $(5,4)$
a. Do the three vectors have the same magnitude?
b. Do the three vectors have the same direction?
c. Look in the Algebra View to see how GeoGebra names vectors. Record below the names of the three vectors according to GeoGebra.
d. Why are the numbers the same?
e. How is a vector similar to a ray?
f. How is a vector different from a ray? (Use the Ray Tool, if necessary.)
