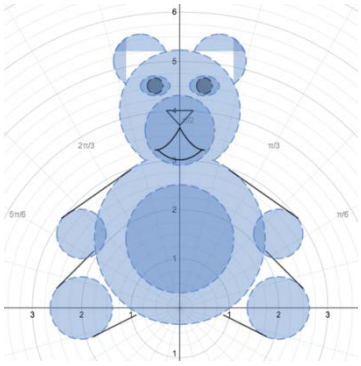
2.6 Polar Curves



Practice Tasks

I. Concepts and Procedures

1. To plot points in polar coordinates, we use a grid consisting of _____ centered at the pole and _____ emanating from the pole.

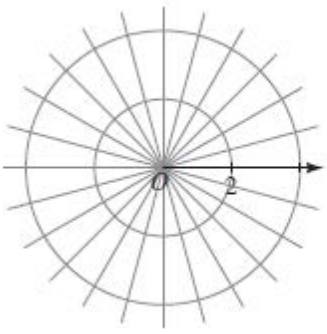
2. To graph a polar equation $r = f(\theta)$, we plot all the points (r, θ) that _____ the equation.

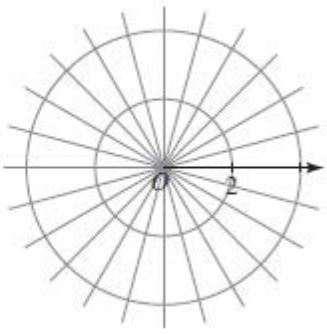
   a. The simplest polar equations are obtained by setting r or θ equal to a constant. The graph of the polar equation $r = 3$ is a _____ with radius centered at the _____.

   b. The graph of the polar equation $\theta = \frac{\pi}{4}$ is a _____ passing through the _____ with slope _____.

   c. Graph these two polar equations below.

$r = \sin(\theta - \frac{\pi}{6})$ $\qquad\qquad$ $r = \sin(\theta + \frac{\pi}{6})$





   d. Explain the results of the two graphs.

3. Match the polar equation with the graphs labeled I–VI.

a. $r = 3\cos\theta$

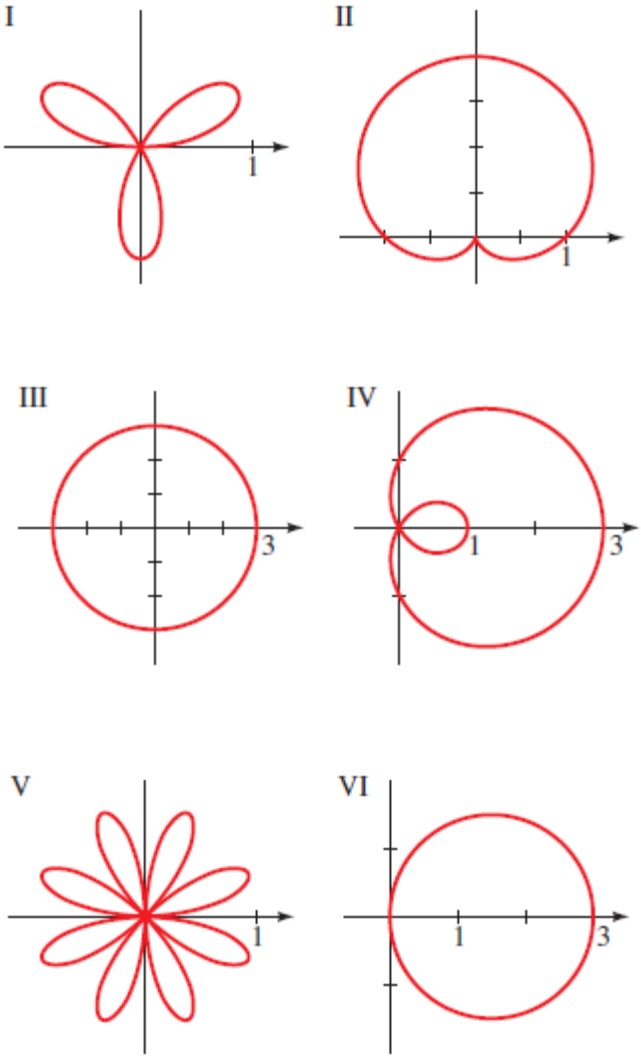
b. $r = 3$

c. $r = 2 + 2\cos\theta$

d. $r = 1 + 2\cos\theta$

e. $r = 3\theta$

f. $r = \sin 4\theta$



3. Sketch a graph of the polar equation, and express the equation in rectangular coordinates.

a. $r = 2$

b. $\theta = -\frac{\pi}{2}$

c. $r = 4\cos 2\theta$

4. Sketch a graph of the polar equation.

a. $r = -2\cos\theta$

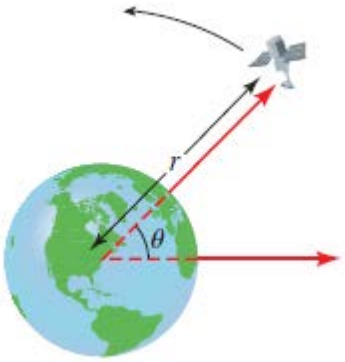
b. $r = -3(1 + \sin\theta)$

c. $r = -\cos 5\theta$

d. $r = \sqrt{3} + \cos(\theta)$

e. $r = 2 + \sec\theta$

II. Problem Solving



1. **Orbit of a Satellite**: Scientists and engineers often use polar equations to model the motion of satellites in earth orbit. Let's consider a satellite whose orbit is modeled by the equation $= \frac{22500}{4-cos\theta}$, where r is the distance in miles between the satellite and the center of the earth and θ is the angle shown in the following figure.
   a. On the same viewing screen, graph the circle $r = 3960$ (to represent the earth, which we will assume to be a sphere of radius 3960 mi.) and the polar equation of the satellite's orbit. Describe the motion of the satellite as θ increases from 0 to 2p.
   b. For what angle θ is the satellite closest to the earth? Find the height of the satellite above the earth's surface for this value of θ.

II. Reasoning

1. Graph the family of polar equations $r = 1 + \sin n\,\theta$ for $n = 1, 2, 3, 4$, and 5. How is the number of loops related to n?

2. Compare the polar equation of the circle $r = 2$ with its equation in rectangular coordinates. In which coordinate system is the equation simpler? Do the same for the equation of the four-leaved rose $r = \sin 2\,\theta$. Which coordinate system would you choose to study these curves?

III. Modeling

1. A Transformation of Polar Graphs: How are the graphs of $r = 1 + \sin(\theta - \frac{\pi}{6})$ and $r = 1 + \sin(\theta - \frac{\pi}{3})$ related to the graph of $r = 1 + \sin\theta$? In general, how is the graph of $r = f(\theta - \alpha)$ related to the graph of $r = f(\theta)$?