2.2 The Elliptical Universe

Practice Tasks

I. Concepts and Procedures

1. An ellipse is the set of all points in the plane for which the _____ of the distances from two fixed points and is constant. The points F_1 and F_2 are called the _____ of the ellipse.

2. The graph of the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with $a > b > 0$ is an ellipse with vertices (_____, _____) and (_____, _____) and foci ($\pm$c , 0) where $c=$ _____ . So the graph of $\frac{x^2}{5^2}+\frac{y^2}{4^2}=1$ is an ellipse with vertices (_____, _____) and (____, ____) and foci (_____, _____) and (_____, _____).

3. Find the vertices, foci, and eccentricity of the ellipse. Determine the lengths of the major and minor axes, and sketch the graph:

   a. $\frac{x^2}{25}+\frac{y^2}{9}=1$

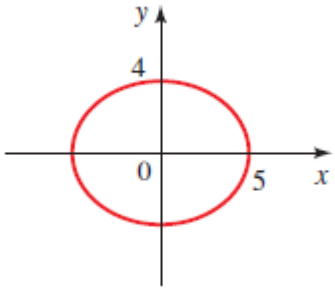
   b. $9x^2 + 4y^2 = 36$

   c. $x^2 + 4y^2= 16$

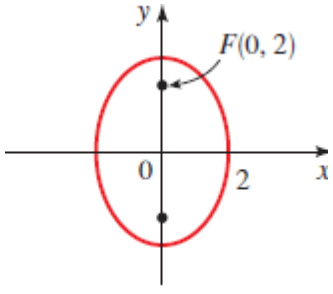
   d. $2x^2 + y^2= 3$

4. Find an equation for the ellipse whose graph is shown:

   a.



   b.



5. Find an equation for the ellipse that satisfies the given conditions.
   a. Foci: $(\pm 4\ ,\ 0)$, Vertices: $(\pm 5\ ,\ 0)$

   b. Length of major axis: 4, length of minor axis: 2, foci on *y*-axis

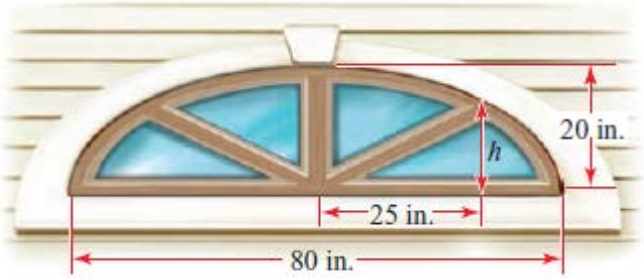
   c. Foci: $(0,\ \pm 2)$, length of minor axis: 6

   d. Length of major axis: 10, foci on *x*-axis, ellipse passes through the point $(\sqrt{5}, 2)$.

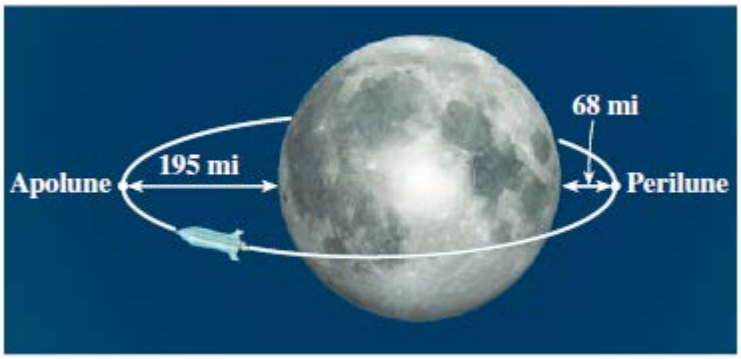
   e. Eccentricity: 0.8; Foci: $(\pm 1.5\ ,\ 0)$

II. Problem Solving

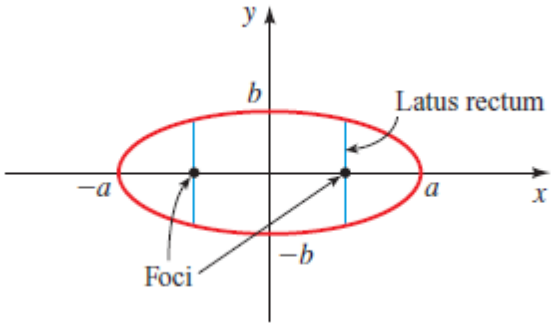
1. **Sunburst Window:** A "sunburst" window above a doorway is constructed in the shape of the top half of an ellipse, as shown in the figure. The window is 20 in. tall at its highest point and 80 in. wide at the bottom. Find the height of the window 25 in. from the center of the base.



2. **Lunar Orbit**: For an object in an elliptical orbit around the moon, the points in the orbit that are closest to and farthest from the center of the moon are called **perilune** and **apolune**, respectively. These are the vertices of the orbit. The center of the moon is at one focus of the orbit. The *Apollo 11* spacecraft was placed in a lunar orbit with perilune at 68 mi and apolune at 195 mi above the surface of the moon. Assuming that the moon is a sphere of radius 1075 mi, find an equation for the orbit of *Apollo 11*. (Place the coordinate axes so that the origin is at the center of the orbit and the foci are located on the x-axis.)



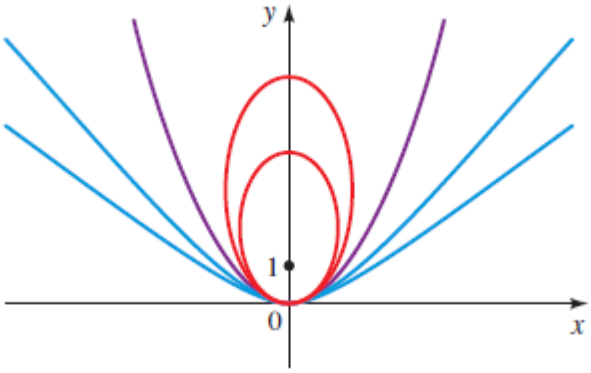
3. **How Wide Is an Ellipse at Its Foci?** A *latus rectum* for an ellipse is a line segment perpendicular to the major axis at a focus, with endpoints on the ellipse, as shown in the figure below. The length of each latus rectum is $\frac{2b^2}{a}$ units.
Write the equation of a horizontal ellipse with center at (3 , 2), major axis length is 16 units, *latus rectum* is 12 units long.



III. Reasoning

1. Determine whether an ellipse represented by $\frac{x^2}{p} + \frac{y^2}{p+r} = 1$ where r >0, will have the same foci as the ellipse represented by $\frac{x^2}{p+r} + \frac{y^2}{p} = 1$.
   Explain your reasoning.

IV. Modeling



1. **A Family of Confocal Conics**. Conics that share a focus are called **confocal**. Consider the family of conics that have a focus at and a vertex at the origin, as shown in the figure.
   a. Find equations of two different ellipses that have these properties
   b. Explain why only one parabola satisfies these properties. Find its equation.
   c. Sketch the conics you found in parts (a) and (b) on the same coordinate axes.
   d. How are the ellipses related to the parabola?