2.1: Crop Circles

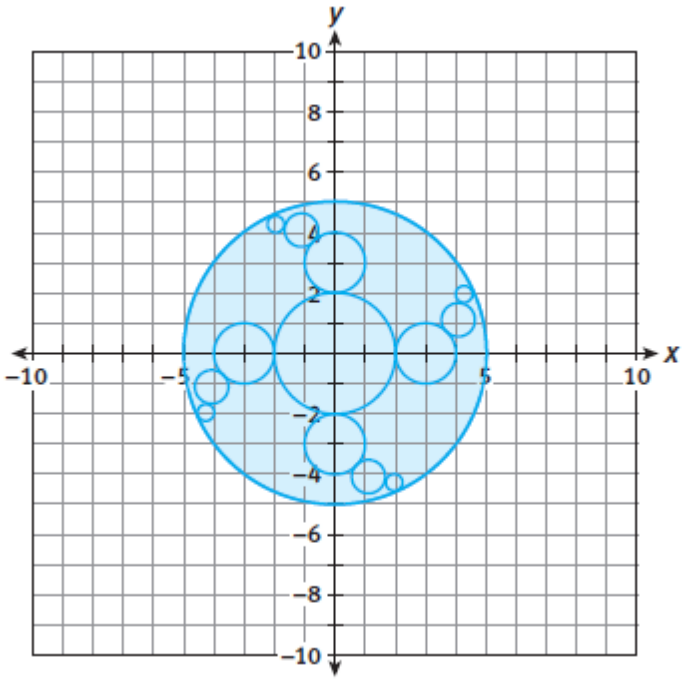
Circles and Degenerate Conics

Crop circles are geometric designs, not always made of circles, created by the flattening of crops in a systematic fashion. They first appeared in the fields of southern England in the mid-1970s. Early circles were quite simple. To this day, crop circles mysteriously appear overnight in fields of wheat, oat, and barley. The crops are flattened; the stalks bent, but not broken. Since 1990, the designs have evolved into complex geometries. However, the term "circle" has stuck.

More crop circles at http://www.livescience.com/26540-crop-circles.html.

Driving Question: How can we create a crop circle design from algebraic formulae?

1. Joan was interested in crop circle designs. She wanted to create one in the field of wheat owned by her father. Joan could not remember how to find the equation of a circle. She decided to use the distance formula to help determine the equations of some circles in a design.
   Use the point (3, 4) to find the radius of the outer circle of the design.



This investigation with the above graphic saved in a blank Desmos file.

2. Label another point (x, y) on the outer circle and list the coordinates here:

3. Use the distance formula, or the Pythagorean theorem and the radius of the circle to write an equation for any point (x, y) on the big outer circle.

4. Find the radius of the smaller circle centered at (0, 0).

5. Use the distance formula and the radius of the circle to write an equation that relates the x and y coordinates of any point on the circle.

6. What is the radius of the crop circle centered at (3, 0)?

7. Write an equation for a circle with radius 1 centered at the origin.

8. How could you transform this equation to represent the small circle centered at (3, 0) in the crop circle diagram?

9. Write an equation for any point (x, y) on the circle centered at (3, 0).

10. Find the radius of the circle centered at (0, 3).

11. Write an equation for the circle centered at (0, 3).

12. Enter equations to complete the crop circle design in Desmos.
Hide the background image and upload your finished graphic here.

Investigation 2

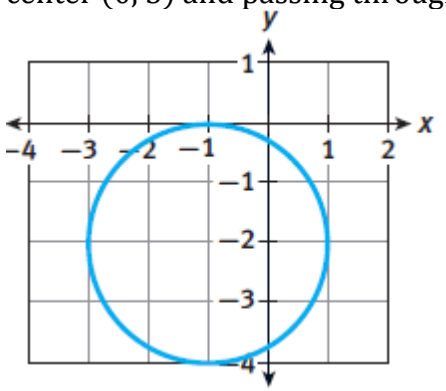
A **circle** is the set of all points in a plane that are equidistant from a fixed point called the center. The standard equation of a circle is $(x \pm h)^2 + (y \pm k)^2 = r^2$ where the center is (h, k) and the radius is r.

13. Suppose that Joan wants to use her design in a wheat field that is 200 meters by 200 meters. She wants the outer circle to be centered in the middle of the field and to have the largest possible circle in proportion to the circle in her design. Write an equation for the outer circle in Joan's crop circle design.

14. Rewrite $\frac{(x-2)^2}{9} + \frac{(y+3)^2}{9} = 1$ in the standard form of a circle, identify the center and radius, and graph.

Another way to write the equation of a circle is $\frac{(x\pm h)^2}{r^2}+\frac{(y\pm k)^2}{r^2}=1$

15. Write the equation of the following circles in the form listed above.
    a. center (-4, 2), radius 12
    b. center (0, 3) and passing through (0, -5)
    c.



16. Graph the following circles and label the center and radius.
    a. $(x-4)^2+(y-1)^2=49$

    b. $(x-3)^2+(y+5)^2=4$

As you have been graphing and identifying geometric properties of the conic sections you have generally been using the standard form of the relation. Each of the conic sections can also be represented by the quadratic relation $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$, where A, B, C, D, E, and F are constants. The values of A, B, C, D, E, and F determine the conic and its properties.

17. Use Desmos or another graphing utility to identify each conic section as a circle, ellipse, hyperbola or parabola, line, or a point.

    a. $9x^2+4y^2-18x+8y-23=0$
    b. $9x^2-4y^2-36x+8y-4=0$
    c. $x^2-y^2-2y-1=0$
    d. $x^2+y^2-6x+10y-47=0$
    e. $x^2+2x-6y+19=0$
    f. $x^2+y^2-6x+8y+25=0$
    g. $x^2+y^2+2x-2y-28=0$

18. What patterns do you notice about the coefficients that can help you identify the conic section when it is in general form?