1.8 "Sine"ing and "Cosine"ing It

Radians and Arc Length

In the previous lesson, you used the similarity of circles, the symmetry of circles, right triangle trigonometry and



proportional reasoning to locate stakes on concentric circles. In this investigation, you will consider points on the simplest circle of all, the circle with a radius of 1, which is often referred to as "*the unit circle*."

Here is a portion of a unit circle—the portion that lies in the first quadrant of a coordinate grid. As in the previous task, *Staking It*, this portion of the unit circle has been divided into intervals measuring $\frac{\pi}{8}$ radians. As in the previous task, find the coordinates of each of the indicated points in the diagram. Also, find the arc length, *s*, from the point (1, 0) to the indicated point.



Javier has been wondering if his calculator will allow him to calculate trigonometric values for angles measured in radians, rather than degrees. He feels like this will simplify much of his computational work when trying to locate the coordinates of stakes on the circles that surround the central tower of the archeological site.

After consulting his calculator's manual, he has learned that he can set his calculator in radian mode. After doing so, he is examining the following calculations.

1. With your calculator set in radian mode, find each of the following values. Record your answers as decimal approximations to the nearest thousandth.

$\sin\left(\frac{\pi}{8}\right) \approx$	$\cos\left(\frac{\pi}{8}\right) \approx$	$\frac{\pi}{8} \approx$
$\sin\left(\frac{\pi}{4}\right) \approx$	$\cos\left(\frac{\pi}{4}\right) \approx$	$rac{\pi}{4} pprox$
$\sin\left(\frac{3\pi}{8}\right) \approx$	$\cos\left(\frac{3\pi}{8}\right) \approx$	$\frac{3\pi}{8} \approx$
$\sin\left(\frac{\pi}{2}\right) \approx$	$\cos\left(\frac{\pi}{2}\right) \approx$	$\frac{\pi}{2} \approx$

2. The coordinates and arc lengths you found for points on the unit circle seem to be showing up in Javier's computations. Why is that so? That is, ...

a) Explain why the radian measure of an angle on the unit circle is the same as the arc length?

b) Explain why the sine of an angle measured in radians is the same as the *y*-coordinate of a point on the unit circle?

c) Explain why the cosine of an angle measured in radians is the same as the *x*-coordinate of a point on the unit circle?

3. Based on this work, find the following without using a calculator:

$\sin\left(\frac{5\pi}{8}\right) \approx$	$\cos\left(\frac{7\pi}{8}\right) \approx$	$\cos \pi \approx$