

1.15 The Amazing Inverse Race

Circular Grids

To entertain themselves on weekends at the archeological dig, Javier has invented a game called “Find My Stake.”

The game consists of drawing two cards, one from a deck of cards that Javier calls “The Angle Specification” cards, and the other from a deck Javier calls the “Location” cards. Based on these two clues, Veronica and Alyce race to locate the position of the stake. The friend who finds the correct location first, wins a prize. Alyce wonders why they need to have two clues. Veronica wonders if two clues will always be enough.

With a partner, play Javier’s game a few times using the two decks of cards that will be provided by your teacher. One of you will draw an “Angle Specification Card.” The other will draw a “Location Card.” See if you can determine the exact location of the stake that is described by the two clues given on the cards. Note that “Angle Specification” cards do not state an angle directly. Rather, they give information about the angle being specified, such as an inverse trig function statement or an equation to be solved. The “Location” cards give additional information to assist you in locating the stake, such as giving the x or y -coordinate of the stake (but not both); or giving r , the distance from the central tower; or perhaps telling the quadrant in which the stake is located.

The archeological site is laid out using both a rectangular grid system and a circular grid system. In the rectangular grid system the horizontal axis represents distances east and west of the central tower, and the vertical axis represents distances north or south of the central tower, the same as on a conventional map. In the circular grid system, concentric circles surround the central tower at equally spaced intervals. Javier has provided both a rectangular grid map and a circular grid map of the archeological site for Veronica and Alyce to use while playing the game. Likewise, your teacher will provide you with both types of grids as you play the game.



Playing the Game

With your partner, play the game at least three times as described above. Each time you play the game, do the following:

- Record the two clues you draw, one from each deck
- Show all work, including calculations, that you do in an attempt to locate the stake
- Choose a rectangular grid or circular grid on which you will record the location of the stake—if you cannot locate the stake exactly, show all possible locations of the stake on the grid; if the clues provide contradictory information, state that a location is impossible to determine
- If possible, determine the location of the stake on both the rectangular and circular grids

Recall that Veronica wondered if two clues would be enough to locate the stake. After playing the game a few times, what do you think?

Analyzing the Game

Examine the clues given to you in the two decks of cards, and then do the following:

- Select a pair of cards that would determine a specific location for the stake—record the clues on the cards and explain why they determine a single, unique location
- Select another pair of cards that would suggest the stake can be located in more than one location—record the clues on the cards and explain why the location of the stake is not uniquely specified
- Select a third pair of cards that give contradictory information—record the clues on the cards and explain the conflict

Repeat these steps a few times until you can answer the following question.

In general, what types of combinations of clues specify a unique location?

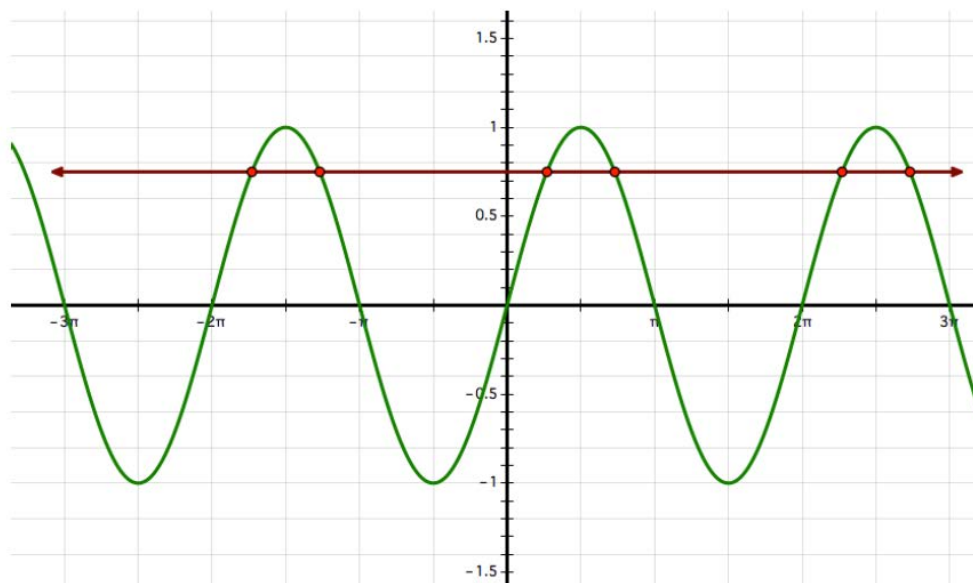
Explaining the Game

For each of the “Angle Specification” cards you had to answer the question, “What angle could fit this given information?” Perhaps you thought about the unit circle or used a calculator to answer this question. For angles of rotation, there are many answers to this question. Therefore, this question—by itself—does not define an inverse trig function.

Suppose you draw this clue from the set of “Angle Specification” cards:

$$\sin \theta = 0.75$$

Using your calculator, $\sin^{-1}(0.75) = 0.848$ radians. However, the following graph indicates other values of θ for which $\sin \theta = 0.75$.



1. Without tracing the graph or using any other calculator analysis tools, use the fact that $\sin^{-1} 0.75 = 0.848$ radians to find at least three other angles θ where $\sin \theta = 0.75$. (Each of these points shows up as a point of intersection between the sine curve and the line $y = 0.75$ in the graph shown above.)

Your calculator has been programmed to use the following definition for the inverse sine function, so that each time you find \sin^{-1} of a number, you will get a unique solution.

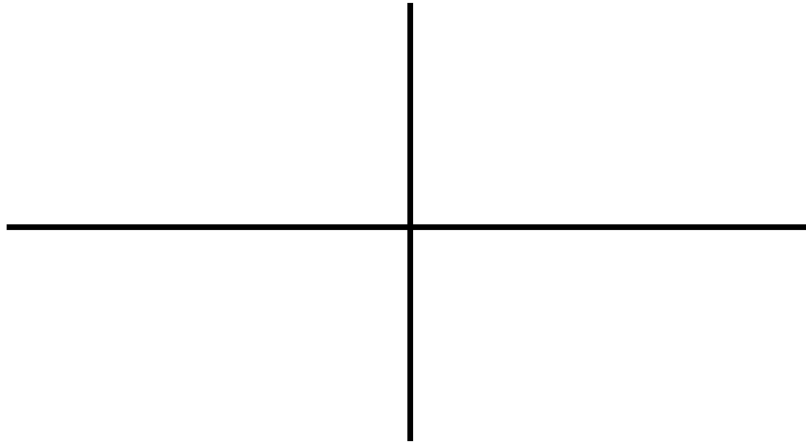
Definition of the inverse sine function: $y = \sin^{-1} x$ means, “find the angle y , on the interval $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, such that $\sin y = x$.”

2. Based on the graph of the sine function, explain why defining the inverse trig function in this way guarantees that it will have a single, unique output.

3. Based on this definition, what is the domain of the inverse trig function?

4. Based on this definition, what is the range of the inverse trig function?

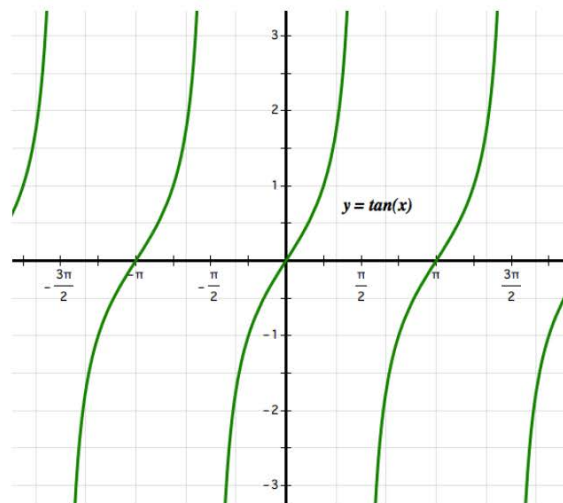
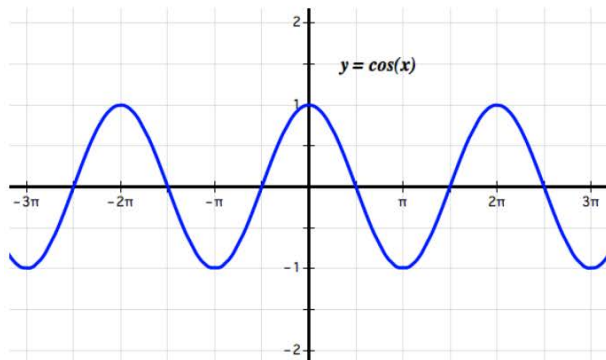
5. Sketch a graph of the inverse sine function.



6. Suppose you draw this clue from the set of “Angle Specification” cards:

$\sin \theta = -\frac{1}{2}$. What is the exact answer to this inverse sine expression: $\sin^{-1} = -\frac{1}{2}$?

Examine the graphs of the cosine function and the tangent function given below. How would you restrict the domains of these trig functions so that the inverse cosine function and the inverse tangent function can be constructed?



Complete the definitions of the inverse cosine function and the inverse tangent function below. State the domain and range of each function, and sketch its graph.

Definition of the inverse cosine function:

Domain:

Range:

Graph:

Definition of the inverse tangent function:

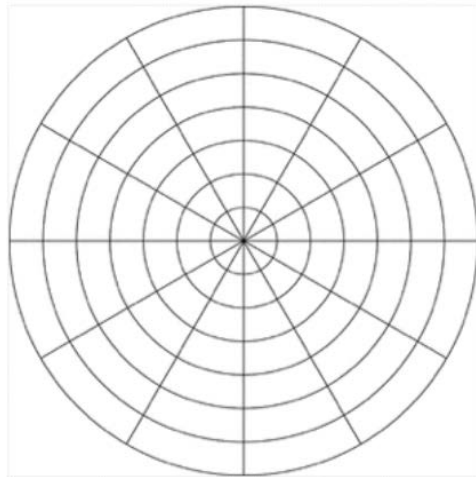
Domain:

Range:

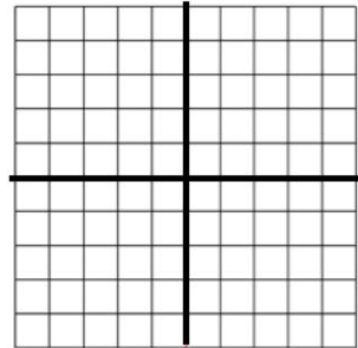
Graph:

Game 1

Location clue:



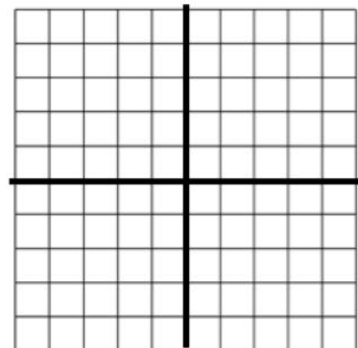
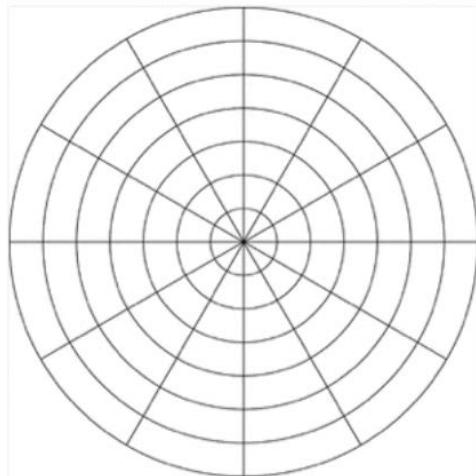
Angle clue:



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Game 2

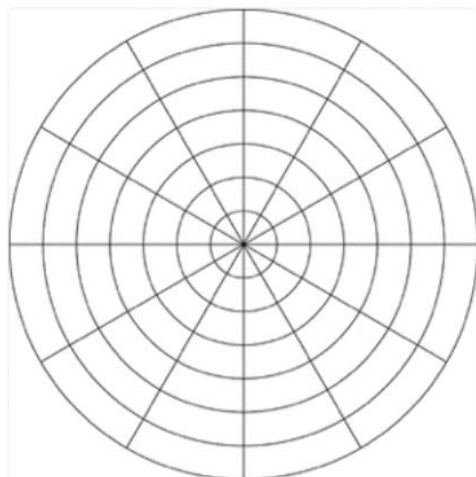
Location clue:



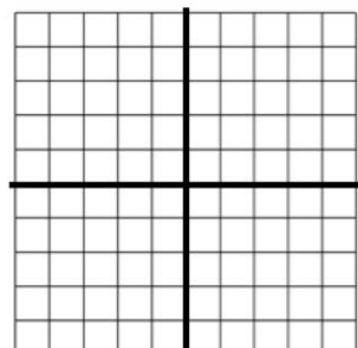
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Game 3

Location clue:



Angle clue:



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