### 1.13 Off on a Tangent

## Extending Definitions

Recall that the right triangle definition of the tangent ratio is:

$$
\tan A=\frac{\text { opposite }}{\text { adjacent }}
$$

1. Revise this definition to find the tangent of any angle of rotation, given in either radians or degrees. Explain why your definition is reasonable.

2. Revise this definition to find the tangent of any angle of rotation drawn in standard position on the unit circle. Explain why your definition isreasonable.

You have observed that on the unit circle the value of sine and cosine can be represented with the length of a line segment.

3, Indicate on the following diagram which segment's length represents the value of $\sin \theta$ and which represents the value of $\cos \theta$ for the givenangle $\theta$.


There is also a line segment that can be defined on the unit circle so that its length represents the value of $\tan \theta$. Consider the length of $\overline{D E}$ in the unit circle diagram
below. Note that $\triangle A D E$ and $\triangle A B C$ are right triangles.

4. Write a convincing argument explaining why the length of segment $\overline{D E}$ is equivalent to the value of $\tan \theta$ for the given angle $\theta$.
5. On the coordinate axes below sketch the graph of $y=\tan \theta$ by considering the length of segment $\overline{D E}$ as $\theta$ rotates through angles from 0 radians to $2 \pi$ radians. Explain any interesting features you notice in your graph.

6. Extend your graph of $y=\tan \theta$ by considering the length of segment $D E$ as $\theta$ rotates through negative angles from 0 radians to $-2 \pi$ radians.
7. Using your unit circle diagrams from the task Water Wheels and the Unit Circle, give exact values for the following trigonometric expressions:

| $\tan \left(\frac{\pi}{6}\right) \approx$ | $\tan \left(\frac{5 \pi}{6}\right) \approx$ | $\tan \left(\frac{7 \pi}{6}\right) \approx$ |
| :---: | :---: | :---: |
| $\tan \left(\frac{\pi}{4}\right) \approx$ | $\tan \left(\frac{3 \pi}{4}\right) \approx$ | $\tan \left(\frac{11 \pi}{6}\right) \approx$ |
| $\tan \left(\frac{\pi}{2}\right) \approx$ | $\tan (\pi) \approx$ | $\tan \left(\frac{7 \pi}{3}\right) \approx$ |

8. Based on the following definitions and your work in this unit, determine how to classify each of the following trigonometric functions.

- A function $f(x)$ is classified as an odd function if $f(-\theta)=-f(\theta)$.
- A function $f(x)$ is classified as an even function if $f(-\theta)=f(\theta)$.
a) The function $y=\sin (x)$ would be classified as an [odd function, even function, neither an odd or even function]. Give evidence for your response.
b) The function $y=\cos (x)$ would be classified as an [odd function, even function, neither an odd or even function]. Give evidence for your response.
c) The function $y=\tan (x)$ would be classified as an [odd function, even function, neither an odd or even function]. Give evidence for your response.

