

1.11 High Tide

Modeling With Trig

Perhaps you have built an elaborate sand castle at the beach only to have it get swept away by the incoming tide.

Spring break is next week and you are planning another trip to the beach. This time you decide to pay attention to the tides so that you can keep track of how much time you have to build and admire your sand castle.



You have a friend who is in calculus who will be going on spring break with you. You give your friend some data from the almanac about high tides along the ocean, as well as a contour map of the beach you intend to visit, and ask her to come up with an equation for the water level on the beach on the day of your trip. According to your friend's analysis, the water level on the beach will fit this equation:

$$f(t) = 20 \sin\left(\frac{\pi}{6}t\right)$$

In this equation, $f(t)$ represents how far the waterline is above or below its average position. The distance is measured in feet, and t represents the elapsed time (in hours) since midnight.

1. What is the highest up the beach (compared to its average position) that the waterline will be during the day? (This is called *high tide*.) What is the lowest that the waterline will be during the day? (This is called *low tide*.)

2. Suppose you plan to build your castle right on the average waterline just as the water has moved below that line. How much time will you have to build your castle before the incoming tide destroys your work?

3. Suppose you want to build your castle 10 feet below the average waterline to take advantage of the damp sand. What is the maximum amount of time you will have to make your castle? How can you convince your friend that your answer is correct?

4. Suppose you want to build your castle 15 feet above the average waterline to give you more time to admire your work. What is the maximum amount of time you will have to make your castle? How can you convince your friend that your answer is correct?

5. You may have answered the previous questions using a graph of the tide function. Is there a way you could use algebra and the inverse sine function to answer these questions. If so, show your work.

Algebraic work for question 3:

Algebraic work for question 4:

6. Suppose you decide you only need two hours to build and admire your castle. What is the lowest point on the beach where you can build it? How can you convince your friend that your answer is correct?