Gödel's Incompleteness Theorems
Knights and Knaves

Knights and Knaves is a logic puzzle due to **Raymond Smullyan** (American mathematician, logician, philosopher, and magician; 1919 -).

These puzzles take place on a fictional island, which we will call Smullyan Island, that consists of two types of citizens: **Knights**, who always tell the truth and **Knaves** who always lie. The goal of the puzzle is to determine the type of citizen each person is based on their statements.

This structure gives a microworld where we can easily explore the truth, falsity or undecidability of statements. In particular we will use this setting to understand two of the most important results in mathematics, the two Incompleteness Theorems of **Kurt Gödel** (Austrian Logician, Mathematician and Philosopher; 1906 - 1978) . Here is an example:

> While on Smullyan Island, you meet two people, Bob and Steve.
> Steve says, “Bob is a knave.”
> Bob says, “Exactly one of us is knight.”
> What are Steve and Bob?

The technique used to solve these types of puzzle is illustrated in the following series of questions.

1. We begin by assuming Steve is a Knight. (We could have started with the assumption that Steve is a Knave, but for our current purposes beginning with the assumption Steve is a Knight is better.)

    a) What does the fact that Steve is a Knight tell us about the truth of his statement? Explain.

    b) What does your answer to Investigation 1a tell you about Bob’s type? Explain

c) What does your answer to Investigation 1b tell you about the truth of Bob's statement? Explain.

d) What do your answers to Investigation 1b - Investigation 1c tell you about the types for both Steve and Bob? Explain

e) Since we are assuming Steve is a Knight, what does your answer to Investigation 1d tell you about Bob's type? Explain.

f) Can your answers to Investigation 1b and Investigation 1e both be true? Explain.

g) Since your answers to both Investigation 1b and Investigation 1e followed from the assumption that Steve is a Knight, what can we conclude about the correctness of our assumption that Steve is a Knight? Explain.

2. Having reached our conclusion in Investigation 1g based on the assumption that Steve is a Knight, we now assume Steve is a Knave.

a) What does the fact that Steve is a Knave tell us about the truth of his statement? Explain.

b) What does you answer to Investigation 2a tell you about Bob's type? Explain

c) What does your answer to Investigation 2b tell you about the truth of Bob's statement? Explain.

d) What do your answers to Investigation 2b - Investigation 2c tell you about the types for both Steve and Bob? Explain

e) Since we are assuming Steve is a Knave, what does your answer to Investigation 2d tell you about Bob's type? Explain.

f) Can your answers to Investigation 2b and Investigation 2e both be true? Explain.

g) Since your answers to both Investigation 2b and Investigation 2e followed from the assumption that Steve is a Knave, what can we conclude about the correctness of our assumption that Steve is a Knave? Explain.

3. Based on your answers to Investigation 1a - Investigation 2g what are the types for Steve and Bob? Explain.

4. Use a reasoning process similar to what you used in Investigation 1 - Investigation 3 to solve the following puzzle:

While on Smullyan Island you meet three people Annie, Betty and Carrie.
Annie says, “Carrie is a Knave”
Betty says, “None of us are Knaves”
Carrie says, “Betty is a Knight.”
What are Annie, Betty and Carrie?

Conditional Statements

One of the most common form of mathematical statements is the **conditional statement**. These are statements of the form “If p then q” where p and q are statements that are either true or false.
We call p the hypothesis and q the conclusion.

Below are several examples of conditional statements.

A. If you are registered for 12 or more credits, then you are considered a full time student at Tunxis Community College.
B. If you live in Connecticut, then you live in the United States.
C. If 6 divides the whole number x, then 3 divides the whole number x.
D. If 3 divides the whole number x then 6 divides the whole number x.
E. If the whole number n is even and greater than 2, then we can find two primes p_1 and p_2 so that $p_1 + p_2 = n$.
F. If I am a Knight then there is gold on Smullyan Island.

With conditional statements, it is important to understand the relationship between the truth or falseness of hypothesis, the conclusion and the conditional statement.

5. For each of the statements A - F, identify the hypothesis and the conclusion.

6. For each of the statements A - F, what do you think it means for the conditional statement to be true? Explain.

7. For each of the statements A - F, what do you think it means for the conditional statement to be false? Explain.

8. For each of the statements A - D, is the conclusion true every time the hypothesis true? Explain.
9. Suppose we believe that the conditional statements E and F are true? What can we conclude when we believe the hypothesis is true? Explain.

Use your answers to Investigation 5 - Investigation 9 to answer the following questions about the general conditional statement “If *p* then *q*;” where *p* and *q* are statements that are either true or false.

10. What do we need to know about *p* and *q* in order for the conditional statement, “If *p* then *q*” to be true?

11. What do we need to know about *p* and *q* in order for the conditional statement, “If *p* then *q*” to be false?

12. If we believe the conditional statement, “If *p* then *q*” to be true, and we believe *p* to be true, what can we conclude about *q*?

13. If we believe the conditional statement, “If *p* then *q*” to be true, and we believe q to be false, what can we conclude about p?

Now let us return to Smullyan Island and explore some of the subtleties of conditional statements on the Island.

14. Can a Knave say, “If I am a Knight, then there is gold on Smullyan Island?” Explain.

15. Can a Knave say, “If I am a Knight, then 1+1=3?” Explain.

16. Can a Knave make any statement of the form, “If I am a Knight, then *q*,” for any statement *q*? Explain.

17. Can a Knight say, “If I am a Knight, then there is gold on Smullyan Island” if there is no gold on the island? Explain.

18. Can a Knight say, “If I am a Knight, then 1+1=3?” Explain.

19. Can a Knight say, “If I am a Knight, then *q*” for any false statement *q*? Explain.

20. Suppose a native of Smullyan Island makes a statement of the form “If I am a Knight, then *q*” for some statement *q*; based on your answers to Investigation 14 - Investigation 19, what can you conclude about the native and the truth or falseness of *q*? Explain.

21. Suppose you meet a native who you believe is a Knight, and the native says, “There is gold on Smullyan Island.” Do you go searching for gold? Explain.

22. Suppose you meet a native who you believe is a Knave, and the native says, “There is gold on Smullyan Island.” Do you go searching for gold? Explain.

23. Can any inhabitant of Smullyan Island ever say “I am not a Knight.”? Explain.

24. Suppose you meet a native who you believe is a Knight, and the native says, “You will believe that I have a pot of gold.” What do you believe? Explain.

25. Suppose you meet a native who you believe is a Knight, and the native says, “You will never believe that I have a pot of gold.” What do you believe? Explain.

26. Suppose you meet a native who you believe is a Knight, explain why you believe whatever the native says.

Our investigation of Gödel’s Incompleteness Theorems will revolve around a native of Smullyan Island making the statement “You will never believe I am a knight.” In order to completely examine this scenario, we need to examine how various reasoners will deal with this situation. Our first case will deal with the situation in which the reasoner, whom we will call Raymond[1], believes he is incapable of making a mistake. That is, if Raymond believes a particular statement, then, as far as he is concerned, the statement is really true. For example, if Raymond ever believes a native of Smullyan Island is a Knight, then because he thinks he is incapable of making mistakes, he concludes that the native really is a Knight and therefore everything that person says is true.

27. Suppose the reasoner, Raymond, believes that he is incapable of making a mistake. He meets a native who says, “You will never believe that I am a Knight.” Suppose Raymond begins with the case that the native is a Knave. What can he conclude from the native’s statement? Explain.

28. Based on your answer to Investigation 27 and the fact that Raymond thinks he is incapable of making mistakes, what does he conclude the native to be? Explain.

29. Explain why your answer to Investigation 28 gives rise to a contradiction.

30. What does your answer to Investigation 29 mean about Raymond’s assumption that the native is a Knave? Explain.

31. Based on your reasoning in Investigation 27 - Investigation 30 what does Raymond now believe the native to be? Explain

32. Based on the native’s statement, “You will never believe I am a Knight”, what is the true status of the native? Explain.

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[1] We need to use a third party as the reasoner because to reach the appropriate conclusions you, the student, need to be “outside” the system.

33. Based on your answers to Investigation 57 - Investigation 62 explain why a reasoner who believes they are always accurate and who meets a native who says, “You will never believe I am a Knight” will eventually become inaccurate.

Most people are not so conceited that they believe they will never make mistakes. However, in this chapter we are interested in conclusions that are logical possibilities, even when they may seem unlikely. The conclusions you made in Investigation 27 - Investigation 33 illustrate this. A more bizarre possibility is illustrated in the next series of questions. For these questions we will assume that our reasoner, Raymond, is what we will call regular. That means he believes that it is impossible for him to believe something and, at the same time, believe that he does not believe it. For example, since Raymond is regular, he believes that it is impossible for him to believe a native is a Knave and, at the same time, believe that he does not believe the native is a Knave.

As before, Raymond goes to Smullyan Island and meets a native who says, ”You will never believe I am a Knight.”

34. Suppose Raymond believes the native is a Knight. Then based on your answer to Investigation 26 what will Raymond also believe? Explain.

35. Since Raymond believes he is regular, what does your answer to Investigation 34 mean about his original belief that the native is a Knight? Explain.

36. Based on your answer to Investigation 35 what will Raymond now believe to be the type of the native? Explain.

37. Based on your answer to Investigation 36 is the native’s original statement true or false? Explain.

38. Based on your answer to Investigation 37, what does Raymond now believe to be the type of the native? Explain.

39. Based on your answers to Investigation 35 and Investigation 37 is Raymond regular? Explain.

The argument you worked through in Investigation 34 - Investigation 39 may seem completely unrealistic, but since the rules of the island hold, this conclusion was forced on us. Most reasoners would, understandably, at this point decide that there is something wrong with the rules of the island. However, because we are assuming that the rules of the island hold, no matter what the results are, we are forced to conclude that, at least theoretically, it is possible for a person to both believe and not believe something. This possibility is essentially what lies at the heart of Gödel’s Incompleteness Theorems.

The Treachery of Images by **René Magritte** (Belgian surrealist painter), Oil on canvas 1929-30

Now that you have an understanding of the rules for Smullyan Island you are almost ready begin your investigation of Gödel's Incompleteness Theorems. However, before that you need to consider conditional statements.

We call a reasoner **inconsistent** if there is some statement *p*, such that the reasoner believes both *p* and its negation, denoted $\sim p$. We therefore call a reasoner **consistent** if there is no statement p such that the reasoner believes both *p* and its negation $\sim p$.

We are now ready to explore Gödel's seminal results using the Knights and Knaves from Smullyan Island. Both results are based on the conclusions we can draw from a native of the Island whose says, "You will never believe that I am a Knight."[2]

Gödel's First Incompleteness Theorem via Knights and Knaves

A consistent reasoner visits Smullyan Island and meets a native who says, "You will never believe that I am a Knight."

40. Suppose the reasoner believes the native is a Knight. Explain why it is reasonable to conclude that the reasoner believes that she believes the native is a Knight.

41. Since the reasoner believes that the native is a Knight, what must she also believe, based on what the native said? Explain.

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[2] It is logically impossible for a native of Smullyan Island to say, "You will never know that I am a Knight" or "You will never correctly believe that I am a Knight." (Smullyan pp. 67-71)

42. Explain why your answers to Investigation 40 - Investigation 41 show that the reasoner will thus never actually believe the native is a Knight since she is consistent.

43. Alternatively, suppose that the reasoner at some point believes that the native is a Knave. What must she also believe, based on what the native said? Explain.

44. Explain why it is reasonable to conclude from your answer to Investigation 43 that the reasoner must also believe that the native is a Knight.

45. Explain why your answers to Investigation 43 - Investigation 44 show that the reasoner will thus never actually believe the native is a Knave since she is consistent.

46. Use your answers to Investigation 40 - Investigation 45 to explain why a reasoner visiting Smullyan Island will never believe that this native is Knight nor that this native is a Knave.

47. Explain why, despite your answer to Investigation 46, we know that the native must be either a Knight or a Knave.

Your answers to Investigation 40 - Investigation 47 illustrate Gödel's First Incompleteness Theorem in the Knights and Knaves setting:

> Theorem A (Gödel's First Incompleteness Theorem in the Knights and Knaves setting (Smullyan, pp. 173-174)). A normal, consistent, stable reasoner of type 1 comes to the island and believes the rules of the island. She meets a native who says: "You will never believe that I am a Knight. Then the reasoner's belief system is incomplete. That is, the reasoner will never believe the native is a Knight and will never believe that the native is not a Knight.

Here is an informal statement of Gödel's First Incompleteness Theorem.

> Theorem B (Gödel's First Incompleteness Theorem). In any sufficiently strong formal system, there are true arithmetical statements that are undecidable within that system.

How does Theorem A relate to Theorem B? Let's begin by parsing the important phrases in Theorem B. The phrase "Sufficiently strong system" refers to a collection of statements where the truth or falseness of most of these statements can be determined from a set of axioms and previously established true statements using the rules of logic. In the Knights and Knaves setting our axioms, or basic assumptions, are

1. Every native of Smullyan Island is a Knight or a Knave.
2. Knights always tell the truth.
3. Knaves always lie.
4. The rules of the Island always hold.

The rules of logic and our assumptions that create a system that is "sufficiently strong". Mathematics, and in particular, number theory, is such a "sufficiently strong" system as well. In fact, if we replace the word believe by provable in assumptions and above and the sentence, "You will never believe that I am a Knight" by "The truth of this sentence can not be proved;" we essentially get the proof of Gödel's First Incompleteness Theorem.

Gödel's Second Incompleteness Theorem via Knights and Knaves

As in Gödel's First Incompleteness Theorem, a reasoner who believes they are consistent and will remain consistent, visits Smullyan Island and meets a native who says, "You will never believe that I am a Knight."

48. Explain why your answers to Investigation 40 - Investigation 42 show that the what the native said is true.

49. Since what the native said is true, what does the reasoner conclude about the native? Explain.

50. What does your answer to Investigation 49 mean that the reasoner believes about the type of the native? Explain.

51. What does your answer to Investigation 50 mean about the truth of the natives statement? Explain.

52. What does your answer mean that the reasoner will have to believe about the native? Explain.

53. Explain why your answers to Investigation 80 and Investigation 82 show that because the reasoner believes that she is consistent, she in fact becomes inconsistent.

Your answers to Investigation 48 -Investigation 53 illustrate Gödel's Second Incompleteness Theorem in the Knights and Knaves setting:

> Theorem C (Gödel's Second Incompleteness Theorem in the Knights and Knaves setting (Smullyan, p. 101)). A reasoner of type 4 visits Smullyan Island and meets a native of the island who says: "You will never believe that I am a Knight." Then, if the reasoner is consistent, she can never know that she is consistent; or stated otherwise, if the reasoner ever believes that she cannot be inconsistent, she will become inconsistent!

Here is an informal statement of Gödel's Second Incompleteness Theorem.

Theorem D(Gödel's Second Incompleteness Theorem). Any sufficiently strong formal system, cannot prove its own consistency within the system. Stated another way, if a sufficiently strong formal system can prove its own consistency then it is inconsistent!

The connections between Theorem C and Theorem D are the same as those between Theorems A and B.

Impact of Gödel's Incompleteness Theorems

Gödel's Incompleteness Theorems have had an enormous impact beyond mathematics. These theorems have entered into the public consciousness in a way few mathematical and scientific results have.

People who quote Gödel's First Incompleteness Theorem recognize that it says something about the limits of mathematical, and hence, human knowledge.

To use Smullyan's ideas we need to have the following assumptions:

1. The reasoner or student needs to believe all tautologies; that is, any proposition that can be "established purely on the the basis of truth table rules for the logical connectives." (Smullyan, p. 43)

2. If, for any propositions p and q, the reasoner believes p and p → q, then (s)he will believe q. (Smullyan, p. 90)

- For any proposition p, Smullyan uses the notation Bp to denote, "the reasoner believes p".
- Using the above notation, condition (2) can be written as [Bp & B(p ⇒ q)] ⇒ Bq.

3. The reasoner believes that if (s)he ever believes both p and p → q, then (s)he will believe q. (Smullyan, p. 90)

- B[Bp & B(p ⇒ q)] ⇒ Bq

4. If the reasoner believes p then (s)he believes (s)he believes p. I think this says that the reasoner is aware of what they believe. (Smullyan, p. 90).

- Bp ⇒ BBp.
- Smullyan calls this condition normal.

5. The reasoner believes that believing p ⇒ (s)he believes (s)he believes p. (Smullyan, p. 90)

- B(Bp ⇒ BBp)

6. The reasoner believes the rules of the island.

These seem reasonable assumptions that we can make about our students. However most students do not understand *modus ponens*, condition (2) above, so we will have to go over this.

- Smullyan calls a reasoner **inconsistent** if for some proposition p, the reasoner believes both p and ~ p. (Smullyan p. 93)
- Smullyan calls a reasoner **type 1** if [s]he satisfies conditions (1) and (2) above. (Smullyan pp. 68-69)
- Smullyan calls a reasoner **peculiar with respect to a proposition *p*** if [s]he believes *p* and believes that [s]he doesn't believe p. (Smullyan p. 81)
- Smullyan calls a reasoner **type 1*** if [s]he satisfies conditions (1) - (2) above and believes that if [s]he ever believes p ⇒ q then whenever [s]he believes *p* then [s]he will believe *q*. (Smullyan p. 83)
- Smullyan calls a reasoner **type 2** if [s]he satisfies conditions (1) - (3) above. (Smullyan pp. 89-90)
- Smullyan calls a reasoner **type 3** if [s]he satisfies conditions (1) - (4) above. (Smullyan p. 90)
- Smullyan calls a reasoner **type 4** if [s]he satisfies conditions (1) - (5) above. (Smullyan p. 90)
- Smullyan calls a reasoner **peculiar** if [s]he believes p and believes [s]he doesn't believe p.
- Smullyan defines a reasoner to be reflexive if for every proposition q, there exists a proposition p such that the reasoner will believe p ≡ (Bp ⇒ q).
- Smullyan calls a reasoner **conceited** if for every proposition p, [s]he believes Bp ⇒ p.
- Smullyan calls a reasoner **modest** if, for every proposition p, whenever [s]he believes Bp ⇒ p [s]he believes p.
- Smullyan calls a reasoner **stable** if for every proposition p, if [s]he believes that [s]he believes p, then [s]he really does believe p.

Smullyan's version of Gödel's Second Incompleteness Theorem

Gödel's Second Incompleteness Theorem states that any system based on arithmetic can not prove its own consistency (within the system). Here is the corresponding theorem in the Knights and Knaves setting:

> <u>Theorem E</u> (Gödel's Second Incompleteness Theorem). Suppose a native of the island says to a reasoner of type 4: "You will never believe that I am a Knight." Then if the reasoner is consistent, [s]he can never know that [s]he is consistent; or stated otherwise, if the reasoner ever believes that [s]he cannot be inconsistent, [s]he will become inconsistent!

Proof. Suppose the reasoner does believe that [s]he is (and will remain consistent). The reasoner reasons: “ Suppose I ever believe that the native is a Knight. Then I’ll believe what [s]he said- I’ll believe that I don’t believe that [s]he is a Knight. But also, if I believe [s]he’s a Knight, then I’ll believe that I do believe [s]he’s a Knight (since I’m normal [condition 4 above]). Therefore, if I ever believe that [s]he’s a knight, then I’ll believe both that I do believe [s]he’s a Knight and that I don’t believe [s]he’s a Knight, which means I’ll be inconsistent. Now I’ll never be inconsistent, hence I will never believe [s]he’s a Knight. [S]He said that I would never believe [s]he’s a Knight, and what he said was true, hence [s]he is a Knight”

At this point, the reasoner believes the native is a Knight, and since [s]he is normal (condition 4 above), he will then know that [s]he believes this. Hence the reasoner will continue: “Now I believe [s]he is a Knight. [S]He said that I never would, hence [s]he made a false statement, so he not a Knight.”
At this point the reasoner believes that the native is a Knight and also believes that the native is not a Knight, and so [s]he is now inconsistent. (Smullyan pp. 101-102.)

Connection to Gödel’s second Completeness Theorem

Here is Smullyan’s description of how the Knights and Knaves setting of Gödel’s Second Incompleteness Theorem (Smullyan, pp. 107 - 111) connects back to mathematics.
The types of systems, S, investigated by Gödel had the following properties:

1. There is a well-defined set of propositions expressible in *S*.
2. *S* has various axioms and logical rules making certain propositions provable in the system. We thus have a well-defined subset of the set of propositions of the system- namely, the set of provable propositions of the system.
3. For any proposition $p \in S$, the proposition that p is provable in S, denoted Bp, is itself a proposition of the system. (Note Bp may or may not be provable in S.)

Systems of type 1, 1*, 2, 3 and 4, are defined in an analogous way to the way we did for reasoners (using the symbolic representations).

Smullyan calls a system, S, a **Gödelian system** if there is a proposition $p \in S$ such that $p \equiv \sim Bp$ is provable in S.
The Knight and Knaves result is a specific example of a G¨odelian system of type 4 and, in fact, Gödel’s Second Incompleteness Theorem proves that all G¨odelian systems of type 4 can not prove their own consistency.
Examples of Gödellian systems of type 4 include:

- Russell and Whitehead’s system in Principia Mathematica.
- First Order Peano Arithmetic

These systems therefore, cannot prove their own consistency.

Smullyan's version of Gödel's First Incompleteness Theorem

Gödel's First Incompleteness Theorem states that any sufficiently strong formal system there are true arithmetical statements that are undecidable in the system. Here is the corresponding theorem in the Knights and Knaves setting:

> Theorem F (Gödel's First Incompleteness Theorem). A normal, consistent, stable reasoner of type 1 comes to the island and believes the rules of the island. [S]he meets a native who says: "You will never believe that I am a Knight. Then the reasoner's belief system is incomplete. That is, the reasoner will never believe the native is a Knight and will never believe that the native is not a Knight.
>
> Proof. First note that since the native said, "You will never believe I am a Knight", the reasoner will believe "the native is a Knight if [s]he doesn't believe the native is a Knight." Stated symbolically:
> if K represents the statement "The native is a Knight", then the reasoner believes $K \equiv\sim BK$.
>
> Suppose the reasoner believes K. Then, being normal, [s]he will believe BK. [S]he will also believe $\sim BK$ (since [s]he believes K and believes $K \equiv\sim BK$ and [s]he is of type 1), hence [s]he will be inconsistent. Therefore, if [s]he is consistent, [s]he will never believe K.

Since the reasoner is of type 1 and believes $K \equiv\sim BK$, the reasoner also believes $\sim K \equiv BK$. Now suppose the reasoner ever believes $\sim K$, then [s]he believes BK. Being stable, [s]he will then believe K and hence become inconsistent (since [s]he believes $\sim K$). Therefore being both stable and consistent, [s]he will never believe $\sim K$.

This is hard to translate into non-symbolic language:

> Suppose the reasoner believes the native is a Knight. Being normal, the reasoner will believe that [s]he believes the native is a Knight. On the other hand, being of type 1, the reasoner will believe what the native says, namely that they will never believe the native is a Knight. Thus the reasoner believes [s]he believes the reasoner is a Knight and believes [s]he doesn't believe the native is a Knight. Since the reasoner is inconsistent, [s]he will never believe the native is a Knight.
> Since the reasoner is of type 1 and believes "the native is a Knight iff [s]he doesn't believe the native is a Knight"; the reasoner also believes "the native is not a Knight iff [s]he believes the native is a Knight." Suppose the reasoner at some point does not believe the native is a Knight; then [s]he believes that [s]he believes the native is a Knight; being stable, the reasoner then believes that the native is a Knight and hence become inconsistent (since [s]he believes the native is not a Knight). Therefore being both stable and consistent, [s]he will never believe the native is not a Knight.

Connection to Gödel's First Completeness Theorem

Here is how I see the Knights and Knaves setting of Gödel's Second Incompleteness Theorem (Smullyan, pp. 107 - 111) connects back to mathematics.

Gödel essentially proved any consistent, normal, stable Gödelian system, S of type 1 must be incomplete. More specifically, if S is a consistent, normal, stable Gödelian system of type 1, and there is a proposition $p \in S$ such that $p \equiv \sim Bp$ is provable in S, then neither p nor $\sim$ p is provable in S.

Since Russell and Whitehead's system from *Principia Mathematica* and *First Order Peano Arithmetic* are normal, stable Gödellian systems of type 4, they are normal, stable Gödellian systems of type 1 and hence if they are consistent, they will contain undecidable propositions.

Printer Version of Gödel's Second Incompleteness Theorem:

From http://rationalwiki.org/wiki/Essay:Godel's incompleteness theorem simply explained

You have a magic printer, when you type a true statement into the computer, it will print the statement out. If you type in a false statement, it will not print it out. For example, if you type in the statement, "Today is Independence Day in the United States" on July 4, the printer prints out "Today is Independence Day in the United States". If you type in the statement, "Today is Independence Day in the United States" on August 4, the printer prints out nothing.

54. Suppose you type in the statement "2 + 2 = 4". What will the printer do? Explain.

55. Suppose you type in the statement "2 + 2 = 5". What will the printer do? Explain.

56. Suppose you type in the statement "The printer can print the statement '2 + 2 = 4'." What will the printer do? Explain.

57. Suppose you type in the statement "The printer can print the statement '2 + 2 = 5'." What will the printer do? Explain.

58. Suppose you type in the statement "The printer cannot print the statement '2 + 2 = 4'." What will the printer do? Explain.

59. Suppose you type in the statement "The printer cannot print the statement '2 + 2 = 5'." What will the printer do? Explain.