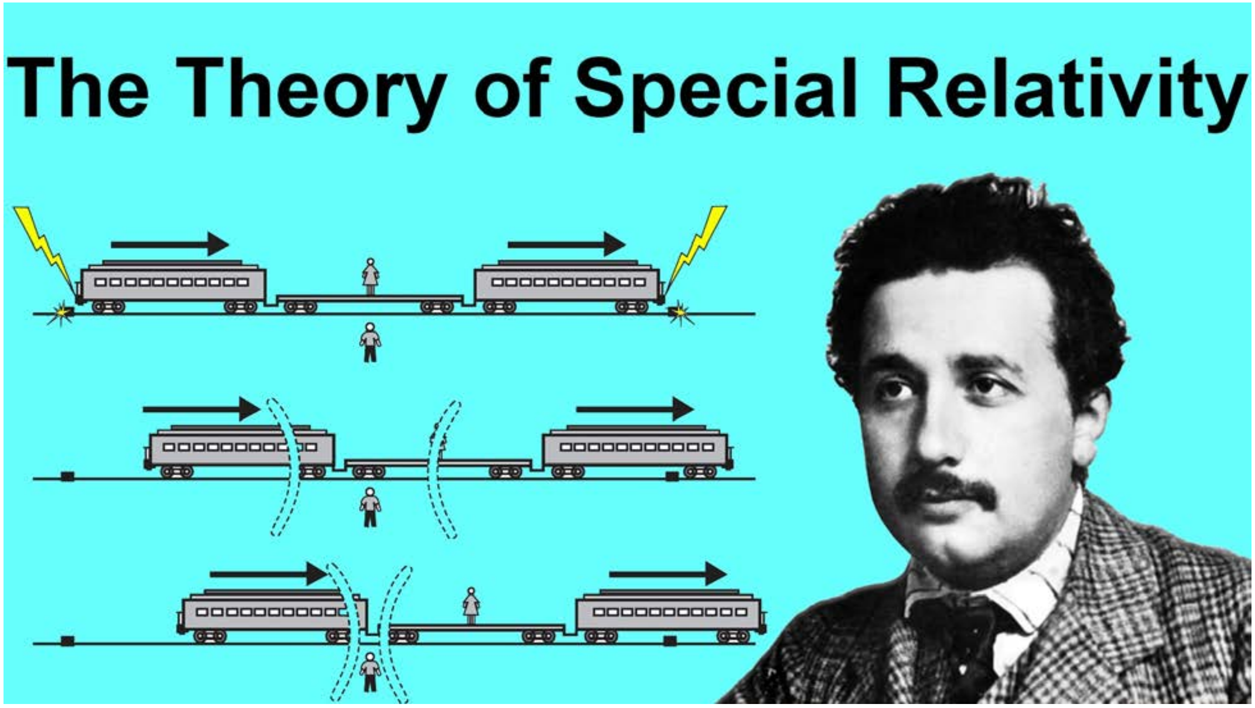
Einstein’s Theory of Special Relativity



1. Classroom Discussion: Are you moving? Not just breathing and having blood flow through your body, but moving in the large.

So wondered the great **Galileo Galilee** (Italian mathematician, physicist, astronomer and philosopher; 1564 - 1642) in his famous *Dialogues Concerning the Two Chief World Systems* of 1632.

Written as a dialogue in the great Socratic style[1] this was, as Galileo explained as part of his dedication of the book to the Grand Duke of Tuscany, a book written so “lovers of truth can draw the fruit of greater knowledge and utility.” For “he who looks higher is the more highly distinguished, and turning over the great book of nature (which is the proper object of philosophy) is the way to elevate one’s gaze.” And where should we point our gaze? To the sky and heavens above: “The constitution of the universe I believe may be set in first place among all natural things that can be known, for coming before all others in grandeur by reason of its universal content, it must also stand above them all in nobility.”

Over 300 years Einstein was still lauding the importance of this book:

> Galileo’s *Dialogue Concerning the Two Chief World Systems* is a mine of information

[1] a genre of prose literary works developed in Greece at the turn of the fourth century BCE, in which. characters discuss moral and philosophical problems.

> for anyone interested in the cultural history of the Western world and its influence upon economic and political development...The leitmotif which I recognize in Galileo's work is the passionate fight against any kind of dogma based on authority. Only experience and careful reflection are accepted by him as criteria of truth. Nowadays it is hard for us to grasp how sinister and revolutionary such an attitude appeared at Galileo's time, when merely to doubt the truth of opinions which had no basis but authority was considered a capital crime and punished accordingly. Actually we are by no means so far removed from such a situation even today as many of us would like to flatter ourselves; but in theory, at least, the principle of unbiased thought has won out, and most people are willing to pay lip service to this principle.[2]
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> **Albert Einstein** (German physicist and author; 1879 - 1955)

2. Classroom Discussion: What are some of the influences "upon economic and political development" that Galileo's work had?

3. Classroom Discussion: Einstein laments that "we are by no means so far removed from such a situation even today." What are some contemporary areas in which we are compelled not to "doubt the truth of opinions which [have] no basis but authority"?

For this, Galileo was brought to trial by the Catholic Church's infamous Inquisition. He was found "vehemently suspect of heresy," placed under house arrest for the remainder of his life, and publication of all of his works were banned. The church's ban on Galileo's *Dialogue Concerning the Two Chief World Systems* was not lifted until 1835!

In his *Dialogues*, Galileo invites the reader to consider some thought experiments whose impact on our understanding of the nature of motion are profound. They offer a theory of relativity for systems in motion.

One of his more famous thought experiments is as follows:

> Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it.
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> With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need to throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction.

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[2] Foreword to Galileo's *Dialogue Concerning the Two Chief World Systems*, University of California Press, 1962 edition.

> When you have observed all of these things carefully (though there is no doubt that when the ship is standing still everything must happen this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than towards the prow even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor under you will be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite.
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> The droplets will fall as before into the vessel beneath without dropping towards the stern, although while the drops are in the air the ship runs many spans. The fish in the water will swim towards the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air.

4. What are the implications of this thought experiment on the issues you considered in Discussion 3? What role could such a thought experiment play on the debate about possible nature of the motions of the earth and of the solar system?

We will extend this thought experiment a bit. Consider a mate standing high up in the crows nest atop the mast of a sailing ship. The ship is moving quite quickly although the sea and air are calm. The mate has a very small ball which will not be affected much by wind resistance and has a nearly perfectly elastic bounce. The mate drops the ball, it falls under the forces of gravity until it hits the deck, bounces back up and is caught by the mate when it returns to the height of her hand.

5. Suppose you were the captain of the ship and were standing by the rail, directly across from the mast. Precisely describe what the path of the ball would look like to you. How far would the ball appear to travel?

6. Now suppose that you were on the beach, watching the ship speed past and the direction of the boat is perpendicular to your line of sight to it. Precisely describe what the path of the ball would look like to you. How far would the ball appear to travel?

7. Did the ball appear to travel the same distance in the two different settings? If so, why? If not, how is this possible?

The situation you have just investigated is a consequence of what is called the *Principle of Galilean Relativity*.

You have probably encountered a related situation before:

8. A fire truck is racing down the street with its sirens blaring. How does the sound of the siren compare to different observers who are:

- Standing on the sidewalk as the truck moves toward them?
- Standing on the sidewalk as the truck moves away from them?
- Driving in the same direction, at the same speed, as the truck?
- Driving in the opposite direction from the truck?

In considering the emergence of non-Euclidean geometry we talked about axioms for a system the logical starting points for the microworld. By changing the Parallel Postulate new geometries were discovered, geometries that were just as consistent as the usual Euclidean geometry we are used to, and which have subsequently been found to be profoundly useful.

Einstein's theory of relativity is among the more revolutionary paradigm shifts in the history of science. It is important to note that the *Special Theory of Relativity*, which we will consider here, relies heavily on non-Euclidean geometry. Indeed, it is no accident that relativity was developed on the heels of significant progress in the study of non-Euclidean geometries. In fact, Einstein's profound results were not the lone work of one genius but involved essential mathematical breakthroughs by important contemporaries of Einstein, including Henri Poincare, Bernard Riemann and Edward Lorenz. While we will not talk about Einstein's general theory of relativity of 1915, it has deep mathematical roots as well. One of the shorter descriptions of this theory is as "the geometric theory of gravitation."[3]

It's also interesting to note that the mathematical subtleties of the implications of Einstein's theory of relativity were not always well received nor well understood by Einstein, who said: Since the mathematicians have invaded the theory of relativity, I do not understand it myself anymore.

Here we will focus on Einstein's special theory of relativity which was developed in 1905. We consider it because it has a profound impact on our question of the limits of knowledge and we can investigate it as a fairly straightforward axiomatic system. Indeed, in his attempt to explain the theories of relativity to the common person in Relativity: The Special and General Theory[4] Einstein begins by discussing axioms:e and we can investigate it as a fairly straightforward axiomatic system. Indeed, in his attempt to explain the theories of relativity to the common person in *Relativity: The Special and General Theory*.[5] Einstein begins by discussing axioms:

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[3] From Wikipedia, http://en.wikipedia.org/wiki/General relativity
[4] R.W. Lawson translation, published by Methuen and Co. Ltd, 1920.
[5] R.W. Lawson translation, published by Methuen and Co. Ltd, 1920.

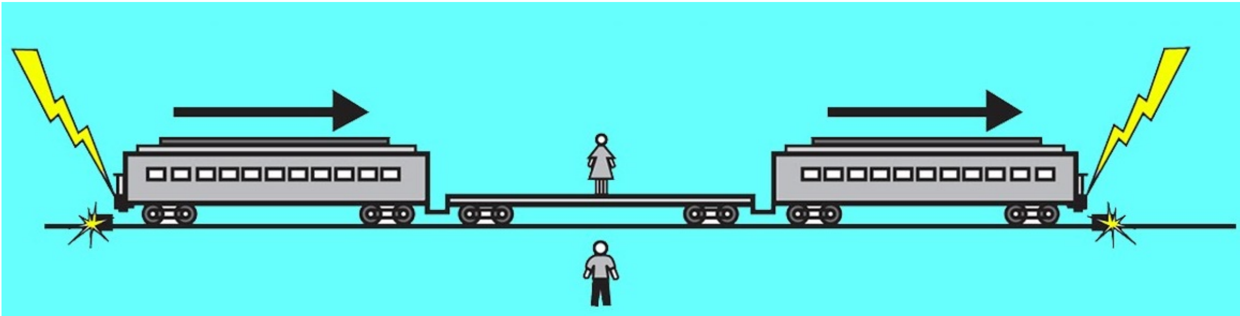
> Geometry sets out form certain conceptions such as “plane,” “point,” and “straight line,” with which we are able to associate more or less definite ideas, and from certain simple propositions (axioms) which, in virtue of these ideas, we are inclined to accept as “true.” Then, on the basis of a logical process, the justification of which we feel ourselves compelled to admit, all remaining propositions are shown to follow from those axioms, i.e. they are proven. A proposition is then correct (“true”) when it has been derived in the recognized manner from the axioms. The question of “truth” of the individual geometrical propositions is thus reduced to one of the “truth” of the axioms. Now it has long been known that the last question is not only unanswerable by the methods of geometry, but that it is in itself entirely without meaning. We cannot ask whether it is true that only one straight line goes through two points. We can only say that Euclidean geometry deals with things called “straight lines,” to each of which is ascribed the property of being uniquely determined by two points situated on it. The concept “true” does not tally with the assertions of pure geometry, because by the word “true” we are eventually in the habit of designating always the correspondence with a “real” object; geometry, however, is not concerned with the relation of the ideas involved in it to objects of experience, but only with the logical connection of these ideas among themselves.

He concludes the section by noting, “we shall see that this ‘truth’ is limited, and we shall consider the extent of its limitation.”

9. Classroom Discussion: Albert Einstein is one of the most famous scientists of all times. His scientific contributions were at their peak as he wrote *Relativity: The Special and General Theory* for general audiences in 1916. What do his efforts to explain these landmark achievements tell you about his view of the common person; the non-scientists? Do you notice some comparison with the goals of the books in this series? Explain.

10. Classroom Discussion: While Einstein continued to do important work in physics throughout his life, he spent considerable amounts of his time later in life exploring the human condition, advocating for peace, promoting education, and engaging in significant philosophical questions - all in very public ways given the renown he had achieved. Find several related quotes by Einstein that you find compelling. Describe them and why you find them compelling.

The next example is directly from *Einstein’s Relativity: The Special and General Theory*.



Suppose a conductor is standing at the exact middle of a long, fast-moving train. A hiker is standing still, next to the tracks, watching the train speed by. Simultaneously flashes of light reach the hiker from bolts of lightning which have hit the very front and very back of the train. The bolts of lightning have left marks on the ground as well and it is later determined that the hiker is standing midway between the lightning strikes.

11. Explain why, from the perspective of the hiker, the bolts of lightning must have hit the train simultaneously.

12. As judged by the hiker, when the bolts of lightning hit the train, where must the conductor have been in relation to the hiker? Explain.

13. In the time it takes for the light from the lightning flashes to travel to the conductor, how has the location of the conductor changed?

14. Which flash of lightning will the conductor see first? Explain.

15. Immediately following this thought experiment, Einstein concludes:

> Every reference-body (co-ordinate system) has its own particular time ; unless we are told the reference-body to which the statement of time refers, there is no meaning in a statement of the time of an event.

Explain this in the context of our thought experiment.

16. What implications does this *relativity of time* have for our discussion of the limits of knowledge? Explain.

We are now in a position to derive the equations governing the relativity of time. They are little more than a careful quantitative investigation of a situation analogous to Galileo's ship experiment only now our focus is on the relative travel of light. The key assumptions we need are two physical axioms that Einstein called postulates. Generally a very empirical science, this chapter of physics resembles mathematics in its derivation and logical structure.

Einstein's postulates are as follows:

- "The same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate"

- "...and also introduce another postulate...namely, that light is always propagated in empty space with a definite [finite] velocity c which is independent of the state of motion of the emitting body."

In analogy with Galileo's ship, consider a rocket ship which is moving quite quickly (say at speed s) although there is no net acceleration in any direction. A small light source which can emit pulses of light is suspended d units above a mirror that reflects light back to the source.

17. Suppose an astronaut was standing next to the light/mirror apparatus. Precisely describe what path the light's travel would appear to take from the perspective of the astronaut. How far would the light appear to travel in this *reference frame*?

18. Use the constant speed of light, which is usually denoted by the letter *c*, to find an equation for the length of time in the reference frame of the astronaut that it appears to take for the light to travel from the source to the mirror and then back to the source. Denote this time by t_1.

19. Now suppose that you were on a planet, watching the rocket ship speed past. Explain why, from an appropriate vantage point[6] the path of the light would look as seen below in your reference frame. Explain why the light would appear to you to travel a distance of $2l$.

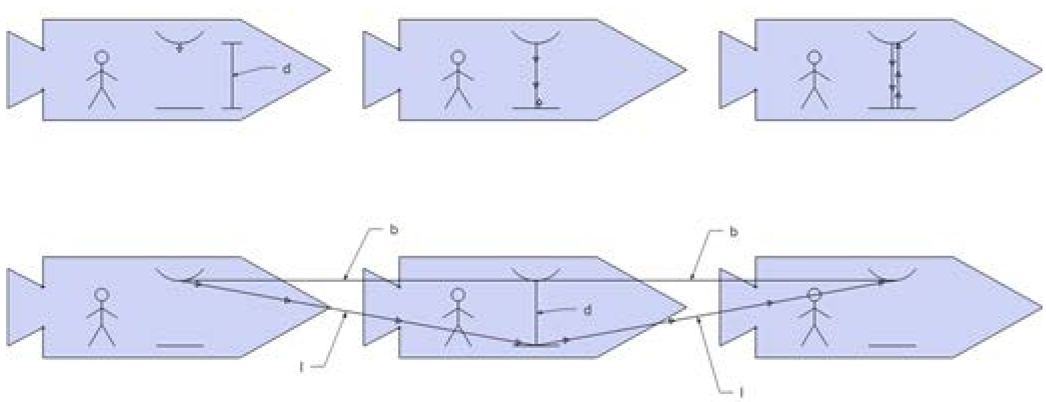


Figure: A beam of light traveling in a rocket ship as pictured at three points in time as seen by an observer moving with the rocket ship (e.g. the astronaut), top, and the same beam of light seen by an observer which is at rest in relation to the rocket ship, bottom.

20. Does the light appear to travel a greater distance to you or to the astronaut? Explain.

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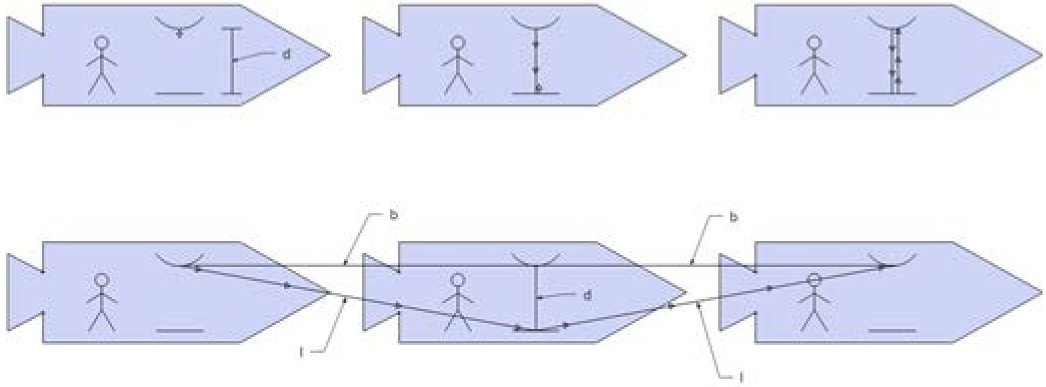
[6] The actual vantage point does not matter as the argument here requires only knowledges of distances so perspective and foreshortening are irrelevant.

21. Because the speed of light is constant in all reference frames, what does Investigation 20 say about the time it took the light to travel from the source to the mirror and back to the source in the two reference frames?



We can now use simple Euclidean geometry to find an exact quantitative relationship between the two different time intervals t_1 and t_2.

22. Use the constant speed of light to find an expression for the length of time, in terms of l, in your reference frame that it takes for the light to travel from the source to the mirror and then back to the source. Denote this time by t_2.

23. Find an expression for the length labeled b in the figure above in terms of t_2 and s, explaining how you obtained this equation.

24. Use the Pythagorean theorem to find a single equation relating l,d, and b.

25. Using results from your investigations above, substitute for each of the parameters l,d, and b in Investigation 24 to obtain an equation involving only the quantities s,c,t_1 and t_2.

26. Solve the equation in the previous investigation for t_2 in terms of t_1. Simplify as much as possible.

27. Use your results to fill in the following table which compares the time intervals t_1 and t_2 at different speeds s

s	t_1	t_2
.01 % c	10 sec	
.1 % c	10 sec	
1 % c	10 sec	
10 % c	10 sec	
50 % c	10 sec	
90 % c	10 sec	
99 % c	10 sec	
99.9 % c	10 sec	
99.99 %c	10 sec	

28. For small speeds, near those that we have been able to historically measure, what do you notice?

29. For large speeds, near the speed of light, what do you notice? What happens if we approach or even exceed the speed of light in this model?

30. In a short essay, describe the implications of some of these results of Einstein to the limits of knowledge.