Deductive Reasoning

Having seen many examples of the limitations of inductive reasoning, we return to deductive reasoning. A theoretical description of deductive reasoning was given in the previous chapter. Remember, deductive reasoning is a type of reasoning where the truth of conclusions follow as logical necessity from previously established premises and the axioms that serve as the foundation of the system.

A useful way to help understand the distinction between inductive and deductive reasoning is to compare corresponding terms in the different areas:

Inductive Reasoning		Deductive Reasoning
Empirical Evidence	↔	Proof
Fits Data	↔	Matches Cause
Hypothesis	↔	Fact
Conjecture	↔	Theorem
Expectation	↔	Logical Conclusion
Reasonable Certainty	↔	Absolute Certainty

In this activity, you will explore some systems which are developed from simple sets of axioms so you can see how deductive systems germinate a rigorous foundation.

Basic Example of Deduction: Introduction to Smullyan Island

Later, in the chapter "Knights and Knaves", you will study in some detail the fictional Smullyan island. It is an island whose cultural code consist of three axioms:

- **Smullyan Island - Axiom 1** All citizens are either knights or knaves.
- **Smullyan Island - Axiom 2** Knights always tell the truth.
- **Smullyan Island - Axiom 3** Knaves always lie.

You find yourself on Smullyan Island, talking to two natives - Don and Sancho. Don says, "We are both knaves."

1. Can Don be a knight? Explain.

2. What can you conclude about the validity of Don's statement? Explain.

3. Deductively establish whether Sancho is a knight or a knave.

What is important to note is that you solved the puzzle of Don and Sancho's types using only logic and the axioms of Smullyan Island's cultural code. Your conclusions are certain as long as the axioms hold.

Basic Example of Deduction: Sudoku Puzzles

Sudoku puzzles are popular puzzles where the goal is to fill in a 9 by 9 grid, which contains some initial clues, with numbers while satisfying each of the following rules:

Sudoku - Rule 1 Each grid square must contain one of the digits 1 through 9.
Sudoku - Rule 2 Every row must contain each of the digits 1 through 9 exactly once.
Sudoku - Rule 3 Every column must contain each of the digits 1 through 9 exactly once.
Sudoku - Rule 4 Every 3 by 3 sub-square must contain each of the digits 1 through 9 exactly once.
Consider the Sudoku puzzle below.

3	2	4			9		7	
		7			2			8
8		6		5		1	2	
9	3		5	1				
			8		4			
				9	3		5	1
	4	3		7		8		9
6			9			4		
	8		6			5	3	7

4. What digit must go in the first row and fourth column? Prove that this is the only possibly digit for this location, explicitly describing what axioms you have used.

5. Where must the 5 in the first row be placed? Prove your result.

6. Where must the 8 in the first row be placed? Prove your result.

7. Where must the 6 in the first row be placed? Prove your result.

Notice that you have, using only the rules of Sudoku - the game's axioms, deductively established the identity of the entire first row for the puzzle to be solved.
8. Where must the 2 go in the lower, right 3 by 3 sub-square? Prove your result.

9. Where must the 6 go in the lower, right 3 by 3 sub-square? Prove your result.

10. Complete the remainder of the lower, right 3 by 3 sub-square, proving your result.

11. Where must the 8 in the eighth row be placed? Prove your result.

12. Where must the 4 in the eighth row be placed? Prove your result.

13. Complete the eighth row, proving that your result is correct.

Notice that at each stage you are proving that your entry must be correct. You are logically employing the rules of the game to make deductive conclusions which definitively establish the identity of certain entries. Notice also that some of your later deductions (moves) rely on previously established results (moves). The proof of each result (move) can be thought of as a theorem. You are building up a larger and larger system with more tools. Indeed, this is precisely one of the ways in which mathematics grows.

45. Solve the Sudoku puzzle, only adding entries when you can deductively establish that this must be the correct entry for this space.

You have now established, for a deductive, permanent fact, that your solution is the one and only solution to this Sudoku puzzle. You have established a Sudoku theorem. The results of mathematics are as final, permanent and definitive as yours. The difference is that mathematics is not constrained