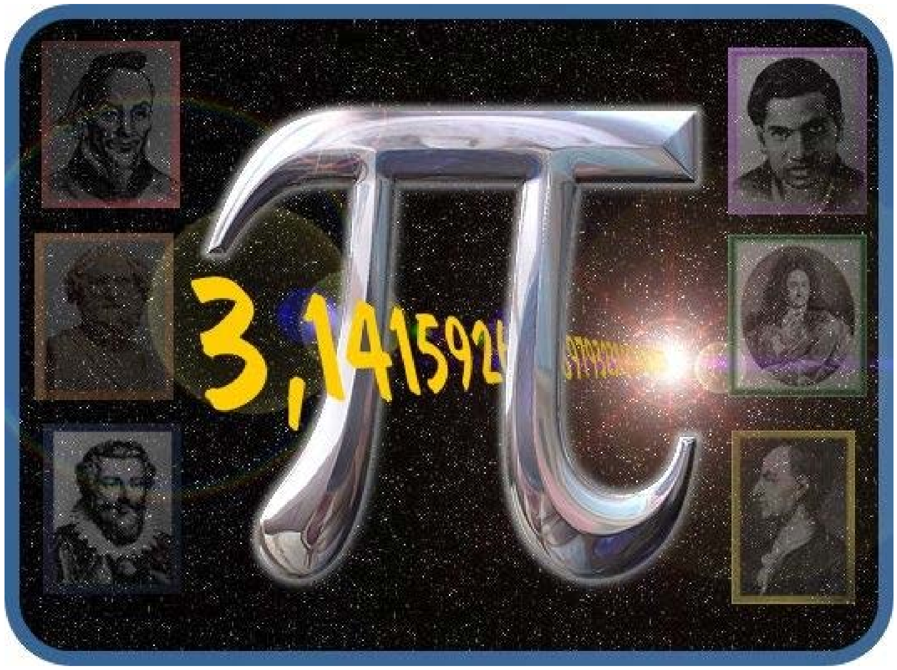


The Mystique of π

We may have brought these creatures into existence (and that is a serious philosophical question in itself) but now they are running amok and doing things we never intended. This is the Frankenstein aspect of mathematics - we have the authority to define our creations, to instill in them whatever features or properties we choose, but we have no say in what behaviors may then ensue as a consequence of our choices.

Paul Lockhart (American mathematician and teacher)

Below we will formally define the number π. It is essential to note that once it is defined, its “behaviors” are no longer under our control. It’s behaviors are under control of the rules of logic within the system in which it was defined. Biblical will, the will of legislators, none of these matter.

There is nothing that gets to be decided once a definition is agreed upon, *π* is what it is and it is impervious to our will. Luckily, it provides the opportunity for wonderful discoveries that can be made about it. As **David Chudnovsky** (American mathematician; 1947 -) says, “Exploring *π* is like exploring the universe.”

The History of π

In 2004 **Daniel Tammet** (English author; 1979 -), who is a savant born with high functioning autism which he has written about in several beautiful and important books,[1] recited correctly from memory the first 22,514 digits of π. For over 5 hours, he rattled off digits of π one after another.[2]

25. What do you know about the number π? Please be as complete and specific as possible. You should include what you know about the number itself, what you know about its properties, where it arises in mathematics, what role it may play in culture, etc.

26. What you know about π, how have you learned it?

27. Are there things about πthat you are curious about or interested in learning more about? If there are specific things, please describe them. If your interest is more general, please describe the nature of this interest. If you are not interesting, please indicate why you are not.

The meme below was recently posted on the Facebook account of *Star Trek*'s **George Takei** (American actor; 1947 -):

> Pi is an infinite, nonrepeating decimal - meaning that every possible number combination exists somewhere in pi. Converted into ASCII text, somewhere in that infinite string of digits is the name of every person you will ever love, the date, time, and manner of your death, and the answers to all the great questions of the universe. Converted into a bitmap, somewhere in that infinite string of digits is a pixel-perfect representation of the first thing you saw on this earth, the last thing you will see before your life leaves you, and all the moments, momentous and mundane, that will occur between those two points. All information that has ever existed or will ever exist, the DNA of every being in the universe, EVERYTHING: all contained in the ratio of a circumference and a diameter.

Are these things really true about π? How could anyone possibly be certain of things like this? There is no way to check all of the digits. No way to search for the unlimited list of different strings that might appear. π is, to many, one of the great mysteries of mathematics.
The fascination with π is nothing new. The table below gives important moments in humanity's search for increasingly better approximations of the value of π. Notice that these efforts span most of the world's cultures over the past four millennia.

---

[1] *Born on a Blue Day*, *Embracing the Wide Sky* and *Thinking in Numbers*
[2] See https://www.youtube.com/watch?v=AbASOcqc1Ss for a selection of a documentary on Daniel which contains footage of this feat.

Mathematician/ Culture	Year	Nationality	Correct Digits
Babylonians	c. 2000 BC		3.1
Egyptians	c. 2000 BC	Egypt	3.1
Bible			3
Plato	c. 380 BC	Greek	3.14
Archimedes	c. 250 BC	Greek	3.14
Hon Han Shu	c. 130		3.1
Ptolemy	150	Greek	3.141
Wang Fau	c. 250		3.1
Liu Hui	263	China	3.14159
Tsu Chhung-Chih	c. 480		3.1415926
Aryabhata	499	Indian	3.1415
Brahmagupta	c. 640	Indian	3.1
Al-Khowarizmi	c. 800	Persian	3.141
Fibonacci	1220	Italian	3.141
Al-Kashi	1429	Persian	14 digits
Nilakantha	c. 1501	Indian	9 digits
Viete	1579	French	9 digits
Romanas	1593		15 digits
Ludolph Van Ceulen	1596	Dutch	20 digits
Ludolph Van Ceulen	1615	Dutch	35 digits
Isaac Newton	1665	English	15 digits
Sharp	1699		71 digits
Machin	1706	English	100 digits
De Lagny	1719	French	111 digits
Matsunaga	1739		50 digits
Vega	1794		136 digits
Rutherford	1824		152 digits

Dase	1844		200 digits
Clausen	1847		248 digits
Lehmann	1853		261 digits
Shanks	1853		530 digits
Lindeman			π transcendental
Ferguson	1945		530 digits
Ferguson	1947		710 digits
Reitwiesner	1949		2,037 digits
Felton	1958		10,020 digits
Shanks and Wrench	1961		100,265 digits
Guilloud and Dichampt	1967		500,000 digits
Guilloud and Boyer	1973		1,001,250 digits
Tamura	1982		16,777,206 digits
Kanada and Tamura	1988		201,326,551 digits
Chudnovskys	1989		1,011,196,691 digits
Takahashi and Kanada	1997		17,179,869,142 digits
Rabinowitz and Wagon	1995		Spigot algorithm discovered
Bailey, Borwein and Plouffe	1995		Hexadecimal digit extraction algorithm discovered

Many of these events are rediscoveries, successes of one culture not readily shared with others - all looking for more digits. Archimedes' method is the method employed in the majority of the events noted through the turn of the seventeenth century. When the number of correct digits does not increase from one line to the next, this is because the approach is new or newly discovered within a culture. Nilakantha was the first to use an approach based on infinite series, using Madhava's series (which is usually known as Gregory's series as it was much better known through its European rediscover). Once this approach was reintroduced in Europe by Newton all of the subsequent results were based on that approach - adding terms from a series representation for π . This accounts for the rapid increase in the number of digits found. Ferguson's first result was the last advance done by hand, his second result done with the aid of a calculator. All subsequent advances were results based on infinite series and computed with the help of electronic computers. In fact, correctly calculating digits of π has long been a way to test computer hardware and software.

Yes, but how in the world does one know that these are really the correct digits of π ? For questions like this, we need to begin to prove things about π .

Defining π

To satisfy the rigors of deductive reasoning we must make careful, unambiguous definitions upon which our mathematical structure will build. The standard definition of the mathematical constant pi, denoted by the Greek letter π , is that it is the unique real number satisfying

$$C = 2\pi r \text{ or } C = \pi d$$

for every Euclidean circle where C is the circumference and r is the radius, or $d = 2r$ is the diameter, of the circle. Equivalently, this means

$$\pi = \frac{C}{2r} \text{ or } \pi = \frac{C}{d}$$

1. Can you explain why the ratio of the circumference to the diameter should be the same for every circle?

This formula for π is one of the most widely taught formulas in school mathematics. It is considered with such repetition that it takes on a nearly self-evident veneer. This is badly misleading. Could you explain why the ratio of the circumference to the diameter is the same for every circle? If not, then the definition for π is entirely nonsensical. This definition presupposes that the ratio of the circumference to the diameter is the same for every circle. For the definition to be logically appropriate one would already have to establish this fundamental result about circles; that is one must prove the theorem that for every Euclidean circle the ratio $\frac{C}{d}$ is the same.

Euclid's *Elements*, so historically important in the development of deductive reasoning and the basis of most high school geometry curricula, is silent on this matter. Nowhere in the 468 propositions that are proven over the course of its 13 books does Euclid consider the circumference of the circle! Ancient mathematicians from many cultures tried to find values for π , clearly suggesting they thought the ratio was constant for all circles. But a definitive understanding of the discovery of proof of this fundamental result has yet to be found.

So how do we make sense of $C = \pi d$? We can begin empirically, as the ancients likely did.

2. Find eight circular objects of significantly different sizes. For each carefully measure the diameter and the circumference. Explain how you determined the diameter.

3. For each of your circles, compute the ratio $\frac{C}{d}$.

4. How close are your ratios to π ? If there are any ratios that are dramatically different than π , remeasure them.

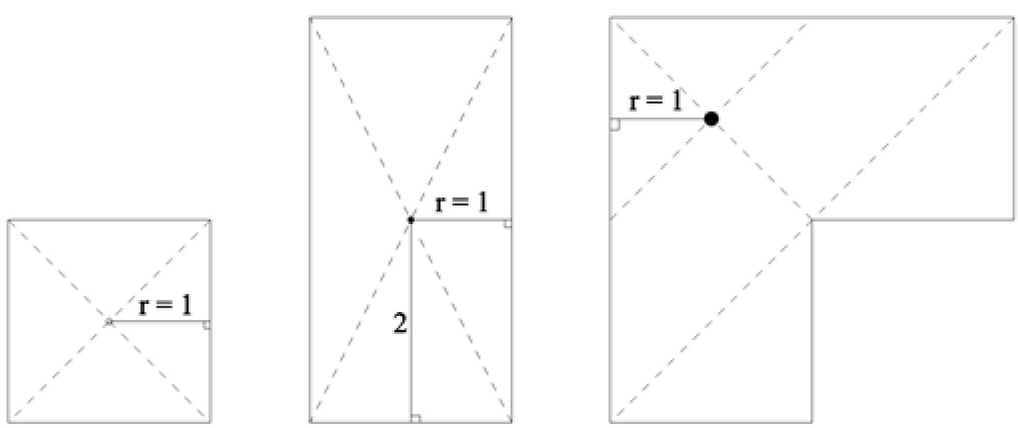
5. Construct a graph for your data points with the horizontal axes starting at $d = 0$ and including the entire range of diameters and the vertical axes the vertical axes starting at $C = 0$ and including the entire range of circumferences. Once you have created your graph, plot your data points. What do you notice about your data?

6. Draw the line $C = \pi d$ on your graph. If you measurements were perfect your data would fall exactly on this line if $\frac{C}{d} = \pi$ for all circles. Is your data within an acceptable range to give you some faith that for all circles? Explain.

Other Shape's πs

Mathematics is not a science. No collection of empirical data is considered sufficient to establish a result. What we require is certainty - a *proof*. One modern way to approach this problem is to look at it more generally:

Driving Question: As we scale other planar shapes does the ratio of their perimeter to some diameter remain constant?



Square, rectangle and right-angled gnomon

7. Choose one of the shapes above to investigate. Find the perimeter of your chosen shape, justifying your work.

Uniformly scale/magnify your chosen shape so that the radius is scaled from 1 to 2. Draw the new shape and find its perimeter, justifying your work.

8. Repeat Investigation 7, this time scaling your originally chosen shape so the radius is scaled from 1 to 3.

9. Repeat Investigation 7, this time scaling your originally chosen shape so the radius is scaled from 1 to 4.

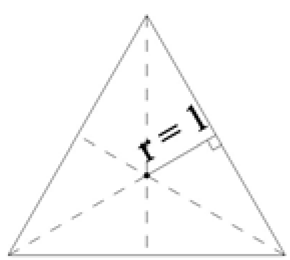
10. Repeat Investigation 7, this time scaling your originally chosen shape so the radius is scaled by a factor of your choice.

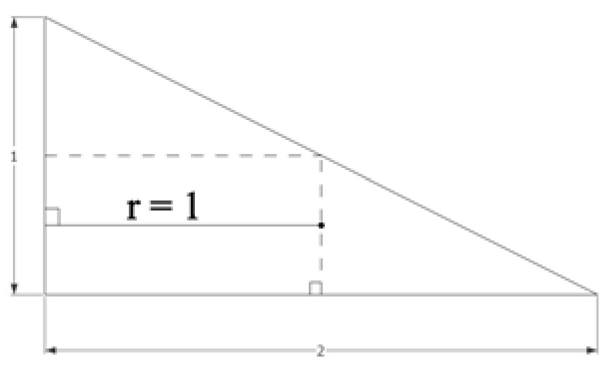
11. Determine the ratio $\frac{perimter}{2 \cdot radius}$ for the shapes in each of the Investigation 7 - Investigation 10. What do you notice?

12. Scale the shape so the indicated length is scaled from 1 to r where r > 0 is any scaling factor. Determine the perimeter, P, and the ratio $\frac{perimter}{2 \cdot radius}$.

13. You now have proven a formula for the perimeter of your chosen shape as it is scaled to any size. Excellent! Isn't it nice to understand why this works and where it came from rather than being asked to mindlessly plug numbers into an arbitrary formula you were given?

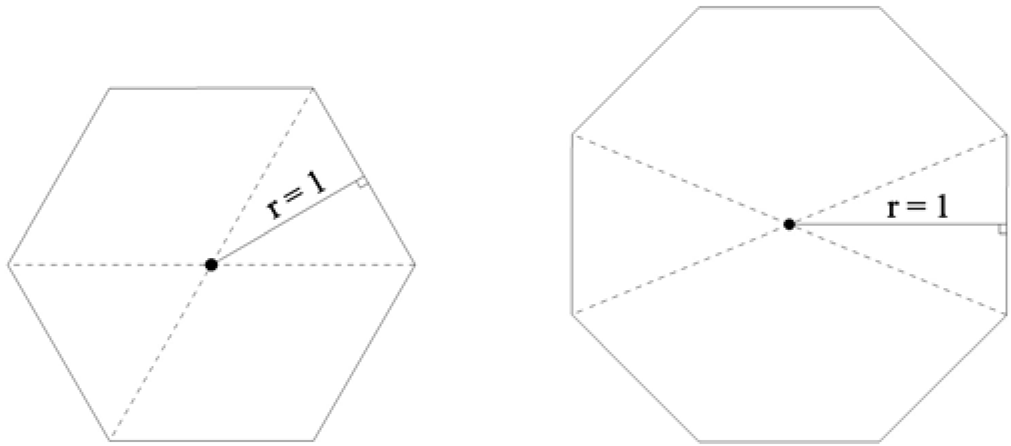
14. Compare and contrast this formula with the formulae C = πd and/or C = $2\pi r$ for circles. Are you surprised that you have found a π -like constant for your chosen shape? Explain.





15. Repeat Investigation 7- Investigation 13 for one of the shapes above

16. Repeat Investigation 7- Investigation 13 for one of the shapes below.



Group Discussion - With other groups, share your results. What have you found? What are the implications for the existence of "other shapes π s" as a way of determining their perimeters?

In fact, it is generally the case that any well-behaved shape will have its own π -like constant ratio that relates the perimeter to the scale r of the shape. This is a fundamental consequence of the notion of *dimension*. The perimeter measures the length of the *boundary* of the shape, the boundary being one-dimensional. Similarly, the length indicated by r is also one-dimensional. As the shape is scaled, both the boundary and the indicated length increase proportionally. Specifically, if we scale by a factor of m the new perimeter is mP and the new indicated length is mr so the ratio of the two is:

$$\frac{mP}{m(2r)} = \frac{P}{2r}.$$

This ratio is unchanged!

Does this prove that $\frac{C}{d}$ is a constant for all circles? Only if one has carefully applied the modern machinery of dimension theory in Euclidean *n*-spaces. One can use calculus as well.[3] But none of these approaches were available to the ancient Greeks. Most teachers and mathematicians, and their books and websites and lectures and class materials, sweep all of this under the rug. Euclid's *Elements* was a paradigm shift, seeking to make mathematical truth eternal by specifying an exact foundation and building deductively upon it. It considered all major areas of mathematical knowledge of the day. Yet there is NO mention of $\frac{C}{d}$ being constant. The ancients

[3] See e.g. p. 543 of *Geometry* (1982) by Moise and Downs.

believed this ratio was constant empirically, but the absence of proof of this fact from the historical record makes clear the difficulty of obtaining a geometric proof in the spirit of Euclid.

Area πs

π also arises in the area formula circle. Why is that so? We could send you out to measure the areas of some circles, but measuring the area of circles is quite hard. Archimedes was a major contributor to our understanding of circular areas and perimeters. The interested reader is referred

For more see Chapter 21.2 of Elementary Geometry from an Advanced Standpoint by E.E. Moise and the paper "Circular reasoning: who first proved that C/d is a constant?" by David Richeson available at http://arxiv.org/ abs/1303.0904.

to the chapter "Areas" in Discovering the Art of Mathematics - Calculus for more on data collection

and the beautiful methods of Archimedes.

Instead, here we'll return to general geometric shapes.

45. Why does the area formula for a circle, $A = \pi r^2$, involve r^2 and not simply r?

46. Do you think the area formulas for our families of shapes should all involve r^2 even though the perimeters involved only r? Explain.

47. For each of the shapes in Investigation 34 - Investigation 38 determine the area of the shape.

48. Determine the ratio $^{Area}{}_r2$ for each shape. Surprised?

45. Scale the original shape so the radius is scaled from 1 to r where r > 0 is any scaling factor. Determine the area, A, and the ratio .

46. You now have proven a formula for the area of your chosen shape as it is scaled to any size. Compare and contrast this formula with the formulae $A = \pi r^2$ for circles. Does it make sense to say that you have found a π-like area constant for your chosen shape? Explain.

47. Repeat Investigation 49 and Investigation 50 for your shape in Investigation 43.

48. Repeat Investigation 49 and Investigation 50 for your shape in Investigation 44.

As with perimeters, this behavior is perfectly natural from the perspective of dimension. The area is a measure of the interior of the shape, the interior being two-dimensional. The length indicated by r is one-dimensional. As the shape is scaled by a factor of m the new indicated length is mr, the area of the new interior is m^2A and the ratio of the new area to the square of the new length is:

.

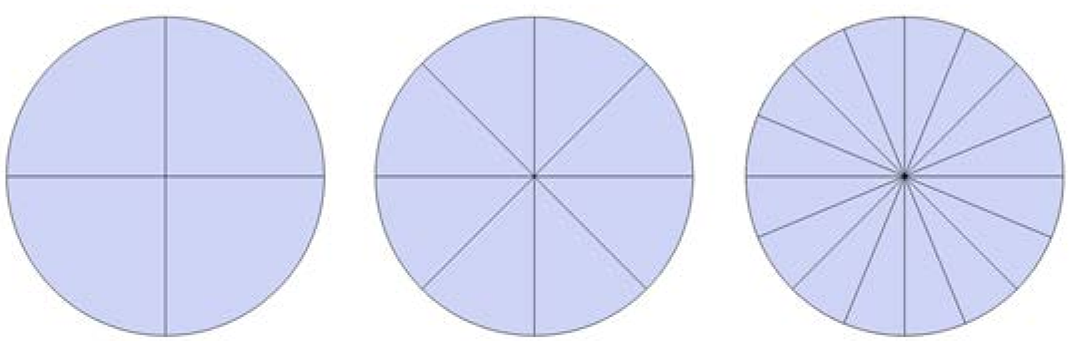
As with perimeters, this ratio is unchanged!
At this stage in the discussion with perimeters we noted that there was no clear evidence in the historic record that the ancient Greeks were able to prove that this ratio is constant. In contrast, the constancy of the ratio for areas of circles is proven as Proposition II of Book XII of Euclid's Elements. This despite the fact that measuring, approximately, circumference is simple while it is remarkably hard to measure area.
In terms of rigorous foundations it would be more reasonable to define π as the ratio which we can prove using basic geometry!
This brings us to a very interesting question: If $\pi \approx 3.14159$ is the constant for the circumference we measure via $C = \pi d$, otherwise the constant is 2π since $C = 2\pi r$), why does this constant have anything to do with the area constant?

53. For each of the three shapes you have chosen to investigate, compare their π-like perimeter constant to their π-like area constant. Are any of these constants the same? Do you see any relationships between the two constants?

Returning to the case of circle, the miracle that the constant in question appears to be π in both cases is something that needs to be proven so we can have certainty.
Enlarged copies of the sectored circles in Figure 3.6 appears in the appendix.

54. Cut out the sectored circle on the left. Try to rearrange the pieces in a way that they create a shape that resembles a shape whose area is easy to determine.
55. Repeat Investigation 54 with the sectored circle in the middle of Figure 3.6. Try to make an arrangement that is similar for each of these circles.
56. Repeat Investigation 54 with the sectored circle on the right of Figure 3.6. Try to make an arrangement that is similar for each of these circles.
57. Suppose you kept sectoring the circle into more and more congruent sectors. Could you continue to arrange the pieces as you did above?
58. If you continued to do this indefinitely, in the limit what would be the resulting shape?
59. Determine the area of this shape in terms of the original dimensions of the circle.
60. Explain how this establishes a direct link between the perimeter constant and the area constant for a circle.

53. Return to the shapes in Figure 3.5. Can you adapt your strategy to give an alternative explanation why the perimeter and area π-like constants for these shapes are equal?



Sectored circles.
This argument has appeared many times historically, including a seventeenth century Japanese text and the works of Leonardo da Vinci (I talian painter, sculptor, architect, musician, mathematician, engineer, inventor, anatomist, geologist, cartographer, botanist, and writer; 1452 - 1519).[1]It was likely known much earlier than this. It is hard to believe that Archimedes was unaware of it.
We now have a real link which unites the circumference constant for circles back to the area constant. This approach is not typical of the type of geometry practiced by Euclid. Rather, it is typical of that practiced by Archimedes, in ways that foreshadowed the development of calculus almost two milennia later. We now, finally, have proof that the perimeter constant is the same as the area constant - our friend π.

3.8.5 Taxicab π

We now consider an alternative geometry. In this new geometry, called Taxicab geometry, distances are measured the way taxicabs d rive city whose streets are laid out so they form a regular, square grid. If you wished to travel by cab from one corner to the opposite corner on the same block a cab would have to drive up one block and then over one block. Hence, the distance between th ese two points is 2. This is a different way to measure distances than in Euclidean geometry. In $\sqrt{}$
Euclidean geometry the distance would be, calculated via the Pythagorean theorem, 2. This is the distance the proverbial crow flies. But a taxicab cannot drive diagonally through the center of a block.
62. Give a precise definition of a circle.
63. On square grid graph paper choose and clearly mark an origin. Find and mark a point whose taxicab distance from the origin is 3 blocks.
64. Find and mark another point whose taxicab distance from the origin is 3 blocks.
65. Repeat Investigation 64.

---

[1] See pp. 17-19 of A History of Pi by Petr Beckmann.

62. Continue to repeat Investigation 64 until you have found all points whose taxicab distance from the origin is 3 blocks. How many such points have you found? In what shape are these points located?
The taxicab geometry we would like to consider is continuous taxicab geometry, where one can follow the streets which make up our grid but can also move along “alleyways” that occur anywhere between the main streets as long as they run parallel to the streets.
63. Utilizing alleyways, find several more points whose taxicab distance from the origin is 3.

64. If you continued using such alleyways, draw the figure which represents all points that are a distance of 3 blocks from the origin.
65. What shape is formed by all of these points?
66. Your shape is formed by all points that are a fixed distance from the origin. Return to Investigation 62. What do we call such a shape?
67. Determine the circumference of your circle of radius 3. Explain carefully how you have determined the circumference.
68. Now draw a circle of radius 2.
69. What is the circumference of this circle?
70. Now draw a circle of radius 1.
71. What is the circumference of this circle?
72. Based on these examples, make a conjecture about the circumference of a circle of radius r in taxicab geometry for positive integers $r > 0$.
73. Prove your conjecture.
74. Having proved the formula for the circumference of circles, determine the value of pi in Taxicab geometry.

3.8.6 Spherical π

You may object that $\pi = 4$ is disingenuous as we are measuring distances so differently in Taxicab geometry. Taxicab geometry is a legitimate geometry. Let us then consider a more natural geometry - the geometry of the surface of the earth. We are, after all, creatures of the earth.

As noted earlier, the relinquishing of Euclid's parallel postulate caused a revolution in geometry in the nineteenth century. Ancient mariners and astronomers did not wait that long to study spherical geometry - they were adept at it close to two milennia earlier.

Find a large spherical object. Lenart spheres are wonderful manipulatives for exploring spherical geometry. If you do not have access, a basketball or other similarly sized spherical object will work fine.

79. On a flat surface, how can you use a piece of string and writing instrument to draw a circle?
80. How do you measure the radius of your circle?

79. You should be able to repeat this same process on your sphere. Draw a circle on the sphere in this way. Does it look like a legitimate circle?
80. Is the string the shortest distance along the surface of the sphere to get from your origin to the circle? If so, does it make sense to call this the radius?
81. Measure your radius and measure the circumference of your circle.
82. Draw another circle on the sphere. Measure its radius and its circumference.
83. Repeat Investigation 84 drawing a circle whose size is much different than those already drawn.
84. Repeat Investigation 84, again trying to draw a circle of a significantly different size.
85. If you let your radius become so large that the circle is in the hemisphere opposite to the "center" where the string is anchored, what happens to the circumference of the circle as the radius gets larger.

86. For each of your circles, compute the ratio of the circumference to twice the radius. This should be our spherical π. Is it? Explain.

3.8.7 But Why 3.14159...?

So in some geometries, shapes have their own πs, often a different π for perimeter than for area. In some they don't.

So the Euclidean circles we are used to have their own π - one you have discovered is the circumference and area constant. But why is this π such a mysterious, complex number? After all, circles are the most symmetric, most perfect of shapes. Why is Euclidean circles' π so esoteric?

The reason is because our way of measuring length is not compatible with the nature of circles. We measure length with straight rulers and square units of area. This is fine for squares and triangles. But our entire measurement apparatus is contrary to the nature of circles. We have a different paradigm, a different perspective than the circle. The operation of the circle must be translated back into our language of lengths and areas. The translation is complex. It is π.

It should not be any other way, should it? π is the crowning jewel of the beauty of the circle perfectly appropriate.

Our race's efforts to understand π is one of the greatest stories of exploration our history holds. Each culture in each age has worked toward understanding its secretes. And their efforts illustrate the evolution of our ways of knowing. What we have found in each case is that π transcends any finite method of measurement.

Each of the formulas below is a formula for π. Each is listed under the name(s) of the mathematician who we believe is the first to discover it.

Nilakantha Somayaji (Indian mathematician and astronomer; 1444 - 1544)

$$\frac{\pi}{8} = \frac{1}{2} - \frac{1}{4^2-1} - \frac{1}{8^2-1} - \frac{1}{12^2-1} - \frac{1}{16^2-1} - \dots$$

Fran¸cois Vi´ete (French mathematician; 1540 - 1603)

$$\frac{2}{\pi} = \sqrt{\frac{1}{2}} \times \sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}} \times \sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}} \times \dots$$

John Wallis (English mathematician; 1616 - 1703)

$$\frac{\pi}{2} = \frac{2\times 2}{1\times 3} \times \frac{4\times 4}{3\times 5} \times \frac{6\times 6}{5\times 7} \times \cdots$$

Isaac Newton (English mathematician and physicist; 1642 - 1727)

$$\frac{\pi}{6} = \frac{1}{2} + \frac{1}{2}\left(\frac{1}{3\times 2^3}\right) + \frac{1\times 3}{2\times 4}\left(\frac{1}{5\times 2^5}\right) + \frac{1\times 3\times 5}{2\times 4\times 6}\left(\frac{1}{7\times 2^7}\right) + \dots$$

M¯adhava (Indian mathematician; circa 1380 - circa 1420), James Gregory (Scottish mathematician; 1638 - 1675) and G.W. Leibniz (German mathematician and philosopher; 1646 1716)

$$frac\pi 4 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$$

William Brouncker (English mathematician; 1620 - 1684)

$$\frac{4}{\pi} = 1 + \frac{1^2}{2 + \frac{3^2}{2 + \frac{5^2}{2 + \frac{7^2}{2 + \dots}}}}$$

Abraham Sharp (English mathematician; 1653 - 1742)

$$\frac{\pi}{6} = \sqrt{\frac{1}{3}} \times \left[1 - \left(\frac{1}{3 \times 3}\right) + \left(\frac{1}{3^2 \times 5}\right) - \left(\frac{1}{3^3 \times 7}\right) + \dots\right]$$

Leonhard Euler (Swiss mathematician; 1707 - 1783)

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$

Srinivas Ramanujan (Indian mathematician; 1887 - 1920)

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{n=0}^{\infty} \left[\frac{(4n)!}{(n!)} \times \frac{[1103 + 26390n]}{(4 \times 99)^{4n}}\right]$$

David Chudnovsky (American mathematician; 1947 -) and Gregory Chudnovsky (American mathematician; 1952 -)

$$\frac{1}{\pi} = 12 \times \sum_{n=0}^{\infty} \left[(-1)^n \times \frac{(6n)!}{(n!)^3 (3n)!} \times \frac{13591409 + 545140134n}{640320^{3n+\frac{3}{2}}}\right]$$

David H. Bailey (American mathematician and computer scientist; 1948 -), Peter Borwein (Canadian mathematician; 1953 -) and Simon Plouffe (Canadian mathematician; 1956 -)
Continued Fraction whose coefficients follow no pattern, like the digits of π

$$\pi = \sum_{n=0}^{\infty} \left[\frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6}\right)\right]$$

Continued Fraction whose coefficients follow no pattern, like the digits of π

$$\pi = 3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1 + \frac{1}{292 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \dots}}}}}}}}$$

Each of these methods involves the infinite.

3.8.8 Current Status of π

Using continued fractions, in 1761 Johann Heinrich Lambert (Swiss mathematician and physicist; 1728 - 1777) was able to prove that π is irrational - it cannot be written as fraction using whole numbers. It is not hard to show[1] that the decimal expansion of any irrational number is nonrepeating. Hence, the decimal expansion of π does not repeat.
Euler and others had long expected that π was even more esoteric, that it was transcendental - not the solution to any polynomial equation with rational coefficients. π resisted for quite some

time until Carl Louis Ferdinand von Lindemann (German mathematician; 1852 - 1939) proved that π was transcendental in 1882. This was seen as a remarkable achievement.

In 1995 two remarkable results were obtained, both spigot algorithms for computing the digits of π. Up to that point all efforts to compute digits of π relied on infinite expressions and sophisticated floating point computer arithmetic. The first, by Stanley Rabinowitz (; -) and Stan Wagon (; -), was a method for computing the digits of π one at a time without any reference to previous digits or need for high precision, floating point arithmetic. All that need be specified in advance was how many digits were desired. The second, by David H. Bailey (American mathematician and computer scientist; 1948 -), Peter Borwein (Canadian mathematician; 1953 -) and Simon Plouffe (Canadian mathematician; 1956 -) allowed any single digit of π to be determined directly without any knowledge of previous digits or much significant calculation. The drawback? The digits are computed not as base ten digits but as base sixteen digits. Why does π submit more readily to base sixteen computations? We do not know.

And what about the claim in Takei's meme that the digits of π contain every possible finite string of digits with the expected frequency? Such numbers are called normal. It is unknown whether π is normal. This remains a great mystery. It is a mystery we may never know the answer to. While the open questions in mathematics far outnumber the questions that have been answered, this is another situation where there is a remarkable silver lining. Mathematicians have proven that the numbers that are normal outnumber those that are not by any measure. Picked randomly, a number is overwhelmingly likely to be normal - to have the remarkable properties claimed in the meme.

3.8.9 You and π

At the outset of this section you were asked some questions about your relationship with π. So how has your relationship with π changed through these investigations?

89. Write a brief essay of one- to two-pages which describes how your relationship with π has changed through the course of these investigations. Some possible topics to include in your essay are:

- What did you learn about π through these investigations?
- What was most surprising to you?
- Were some of the questions you had at the outset answered by your investigations?
- Are you more or less curious about π having completed these investigations?
- Does π make more or less sense to you having completed these investigations? Is this good or bad?

---

[1] See Discovering the Art of Mathematics - The Infinite.