Zeno’s Paradoxes of Motion

There are many famous paradoxes that arise in settings involving the infinite - problems that have confounded philosophers, mathematicians, scientists, writers, and thinkers for thousands of years. Two more are given here. The first is one of the famous paradoxes of the Zeno of Elea (Greek Philosopher; circa 490 B.C. - circa 420 B.C.).

Zeno's paradoxes have puzzled, challenged, influenced, inspired, infuriated, and amused philosophers, mathematicians, and physicists for over two millennia. The most famous are the arguments against motion described by Aristotle in his *Physics*

1. When you’re done working on this investigation you might like to leave the room you’re currently working in. How far is the door from where you are sitting now?

2. Before you can reach the door to leave, you must cover half the distance to the door. How far is this?

3. However, to get halfway to the doorway, you have to cover half of that distance first, right? How far is this?

4. And before you cover the distance from Investigation 3, you must cover half of that distance first, right? How far is this distance from where you are sitting?

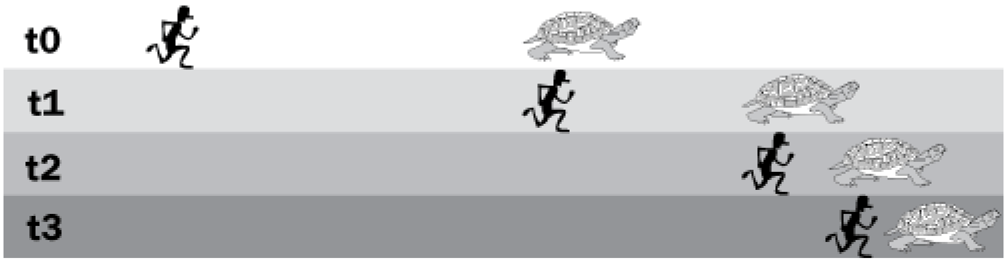
5. Can the reasoning in Investigation 4 be continued? For how long?

6. Before you can leave the room, every single one of the stages described in Investigation 5 must first be completed - all infinitely many of them. What does this seem to suggest about your ability to leave the room?

7. What do these investigations seem to tell you about any kind of motion at all?

8. Is this realistic?

This “forever trapped with my mathematics book” paradox is a version of one of Zeno’s paradoxes referred to as the Zeno’s dichotomy.



Another of Zeno's paradoxes is known as Achilles and the Tortoise. In this paradox the swift Achilles allows a tortoise a half lap lead in a one lap race.

9. As Achilles races to the halfway point, what will the tortoise be doing?

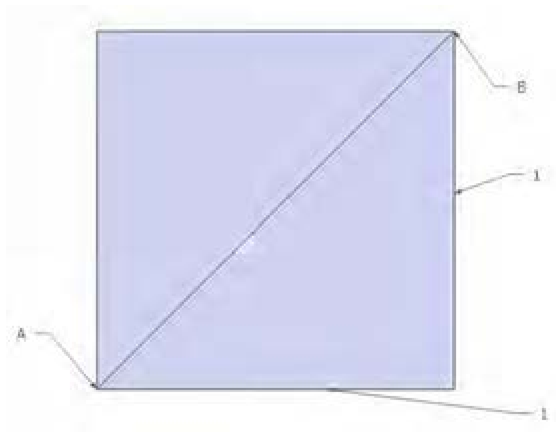
10. As Achilles races from the halfway point to the tortoise's new location, what will the tortoise be doing?

11. In analogy to Zeno's dichotomy, show how the idea of Investigations 9-10 can be repeated indefinitely, suggesting that Achilles will always be behind the tortoise.

12. Does this seem realistic?

13. Investigation 7 and Investigation 11 show why these examples of Zeno's are considered paradoxes. Can you think of any way out of these paradoxes?

Non-Convergence in Measure



Here's another paradox. Consider the unit square above, each of whose sides are 1 unit long. We would like to travel from corner A to corner B.

14. Clearly the most efficient way to get from A to B is a straight line, the diagonal $\underline{AB}$ which has been drawn in as indicated. Find the length L of this diagonal.

15. Travel from A to B by moving vertically one unit and then horizontally one unit. Does this seem to be an efficient way to travel? What is the length of this path?

16. Travel from A to B by alternatively moving vertically and then horizontally in one-half unit increments. Does this seem like it will be a more efficient way to travel? Draw in this path and find its length.

17. Travel from A to B by alternatively moving vertically and then horizontally in one-quarter unit increments. Does this seem like it will be a more efficient way to travel? Draw in this path and find its length.

18. Travel from A to B by alternatively moving vertically and then horizontally in one-eighth unit increments. Does this seem like it will be a more efficient way to travel? Draw in this path and find its length.

19. What do you notice about the lengths of the paths in Investigation 15 - Investigation 18? Will this pattern continue indefinitely?

20. As the length of the vertical and horizontal increments get smaller and smaller, how do the paths compare to the diagonal path $\underline{AB}$? How do the lengths of these paths compare? Can you explain this paradox?
21. Closing Essay - Working with infinite things does not always work out as we expect. In a brief essay, describe your thoughts and feelings about the infinite as this chapter closes.