
# Really Big Numbers

> Behold a Universe so immense that I am lost in it. I no longer know where I am. I am just nothing at all. Our world is terrifying in its insignificance.
>
> **Bernard de Fontenelle** (French Author; 1657 - 1757)

## Large Numbers and the Sand Reckoner

Mathematics has always been used as a tool to describe the world we live in. Although this aspect is far from its sole purpose, it is important. Much of this importance arises from the ability of mathematics to answer quantitative questions: how big, how high, how heavy, how strong, etc.

In a quantitative sense, the infinite must transcend everything that is finite. In terms of numbers, it must be larger than any finite number no matter how large. If we are to appreciate the infinite, a good place to start is by appreciating large numbers.

### Investigation

To get a better sense of the fairly large, let us begin with something we believe we know well: time. In the first five problems you are asked to estimate various quantities. First reactions are what is desired, so you should make these estimates without performing any calculations.

1. Quickly estimate how long you think one thousand seconds are in a more appropriate measure of time.

2. Quickly estimate how long you think one million seconds are in a more appropriate measure of time.

3. Quickly estimate how long you think one billion seconds are in a more appropriate measure of time.

4. Quickly estimate how long you think one trillion seconds are in a more appropriate measure of time.

5. Quickly estimate the number of seconds you have been alive.

6. Precisely determine how long one thousand seconds are in a more appropriate measure of time.

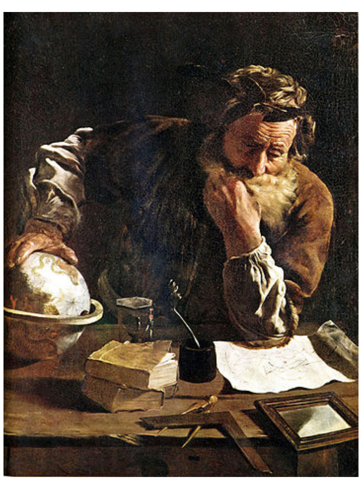
7. Precisely determine how long one million seconds are in a more appropriate measure of time.

8. Precisely determine how long one billion seconds are in a more appropriate measure of time.

9. Precisely determine how long one trillion seconds are in a more appropriate measure of time.

10. What do Investigations 6-9 tell you about the relative size differences between thousands, millions, billions and trillions?

## The Sand Reckoner



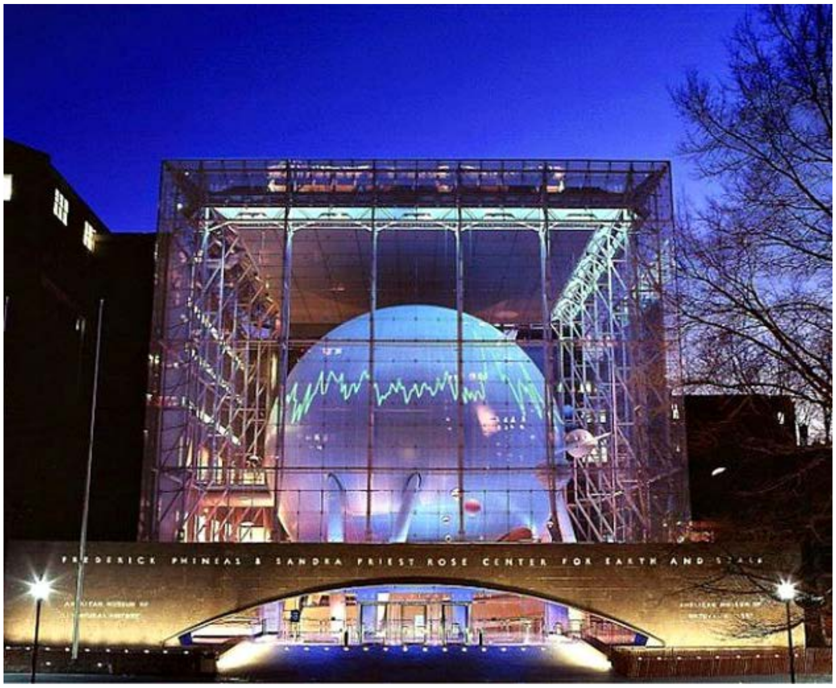
In early times quantifying large magnitudes was difficult. Such a task was considered in detail by the great **Archimedes** (Greek Mathematician, Physicist, Astronomer, Engineer and Inventor; circa 287 BCE - circa 212 BCE) in the *Sand Reckoner*[1]:

There are some, King Gelon, who think that the number of the [grains of] sand is infinite in multitude; and I mean by the sand not only that which exists about Syracuse and the rest of Sicily but also that which is found in every region whether inhabited or uninhabited... But I will try to show you by means of geometrical proofs, which you will be able to follow, that, of the numbers named by me and given in the work which I sent to Zeuxippus, some exceed not only the number of the mass of sand

[1] In several of Archimedes' great contributions we find germs of ideas that lead to the development of calculus almost two millennia later.

equal in magnitude to the earth filled up in the way described, but also that of a mass equal in magnitude to the universe.

Let us update Archimedes somewhat and find a number so large that it exceeds the number of molecules in the entire *observable universe*.




The Hayden Sphere at the American Museum of Natural History in New York City; A powerful vehicle for learning about the relative scales of things great and small.

Of course, we must base our calculation on some theory of the universe.[2] The prevailing scientific theory, the so-called *Big Bang Theory*, suggests that the universe began as a giant explosion some 15 billion years ago. According to Einstein's *Theory of Relativity*, the maximum velocity that matter can travel is the ***speed of light***: approximately $2.9979 \times 10^8 \frac{m}{sec}$. The ***observable universe***, the region from which light can reach us, is therefore a very large sphere. How big might this sphere be? Our figure for the age of the universe is in years, so we would like to convert the speed of light to a yearly distance.

As there are 60 seconds in a minute we can convert the speed of light to a distance per minute as follows:

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[2] Here we will ignore a few more sophisticated *cosmological* issues that complicate the discussion. While these may change the actual calculations, it is in only minor ways that do not fundamentally alter our ability to concisely quantify an upper bound for the size of the observable universe.

$$\text{Speed of light} \approx \left(2.9979 \times 10^8 \frac{m}{\cancel{sec}}\right)\left(60 \frac{\cancel{sec}}{min}\right) = 1.79874 \times 10^{10} \frac{m}{min}.$$

Notice that the speed of light is now given in meters per minute. There is no reference to seconds,
for these dimensions have canceled. We leave it to you (in Investigation 16) to show that in meters per second the speed of light is:

$$\text{Speed of light} \approx 9.45 \times 10^{15} \frac{m}{year}.$$

Now, if the universe expanded for 15 billion years at the speed of light, the result would be a spherical observable universe whose radius is:

$$r = (15 \times 10^9 year)\left(9.45 \times 10^{15} \frac{m}{\cancel{year}}\right) \approx 1.42 \times 10^{26} m.$$

If this observable universe has the radius calculated above, we leave it to you (in Investigation 17) to show it has a volume of:

$$V \approx 120 \times 10^{79} m^3.$$

How many molecules could we pack into such a volume? What we know about the distribution of matter in the universe suggests that such a volume packed with lead would safely overestimate the true number of molecules in the universe.[3]

So we need to figure out how many molecules there are in a cubic meter of lead. The requisite data for lead is a molecular mass of $207 \frac{g}{mole}$ and a density of $11.3 \frac{g}{cm^3}$. Combined with *Avagadro's number*, $6.022 \times 1023 \frac{molecules}{mole}$, we calculate:

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In other words, what this last calculation shows, is that each cubic meter of lead contains about $3.29 \times 10^{28}$ molecules. We have about $1.20 \times 10^{79}$ cubic meters to fill up. So if we filled up the entire observable universe we've been considering with lead, the total number of molecules would be about

$$(1.20 \times 10^{79} m^3)\left(3.29 \times 10^{28} \frac{molecules}{m^3}\right) \approx 3.95 \times 10^{107} molecules.$$

This is not "much more" than a **googol** of molecules!

## Large Numbers and Everyday Things

[3] In contrast to our intuition, a cubic meter of aluminum consists of more than twenty times as many molecules as a cubic meter of lead. Can you find out why? And what is the most "molecular" element so we have an honest upper bound?

11. Compute the number of seconds in one day.

12. Compute the number of seconds in one year.

13. Use Investigation 11 and Investigation 12 to precisely determine, within a hundred thousand seconds or so, the number of seconds you have been alive.

14. How close were your estimates in Investigations 1-4 to the actual results in problems Investigations 6-9 and Investigation 13?

15. What does your answer to problem Investigation 14 tell you about your fluency with large numbers?

16. Show calculations that the speed of light in meters per second is, as stated in the text:

$$\text{Speed of light.} \approx 9.45 \times 10^{15} \frac{m}{year}$$

17. Show calculations that the volume of sphere of radius $r \approx 1.42 \times 10^{26} m$ is given by

$$V \approx 1.20 \times 10^{79} m^3.$$

Using newspaper reports, televised news reports, the Internet, or other source in your library if necessary, find several specific items whose costs or budgets are. Cite the name and date of your source.

18. ... in the thousands of dollars.

19. ... in the millions of dollars.

20. ... in the billions of dollars.

21. ... in the trillions of dollars.