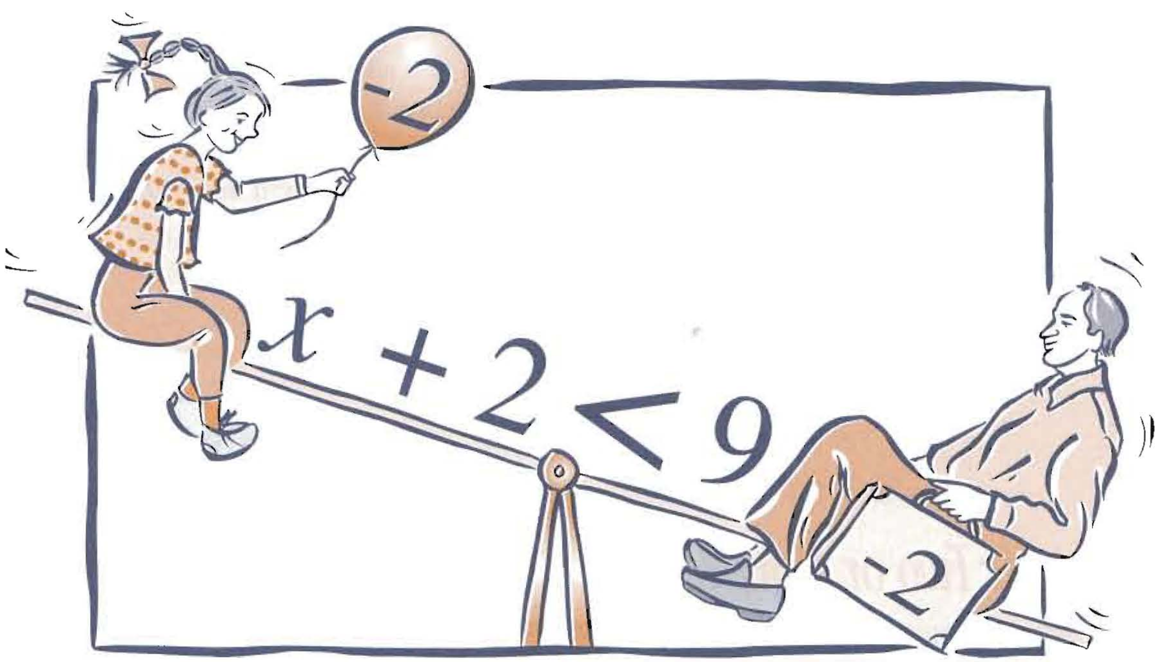

My Simplest Inequality

In *Homework 2: Investigating Inequalities,* you started with a true inequality involving numbers and explored which operations you could do to both sides that would result in another true inequality.

When inequalities involve variables, we want to know whether the operation produces an **equivalent inequality.** As with equations, two inequalities are called *equivalent* if any number that makes one of them true will also make the other true.

For example, the inequalities $x + 2 < 9$ and $2x + 4 < 18$ are equivalent because, in both cases, the numbers that make them true are precisely the numbers less than 7. For instance, substituting 5 for x makes both true but substituting 10 for x makes both false. (That is, $5 + 2 < 9$ and $2 \cdot 5 + 4 < 18$ are both true, and $10 + 2 < 9$ and $2 \cdot 10 + 4 < 18$ are both false.)

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Part I: One Variable Only

If an inequality has only one variable, you can often find an equivalent inequality that essentially gives the solution. For instance, by subtracting 2 from both sides of the inequality $x + 2 < 9$, you get the equivalent inequality $x < 7$. This tells you that the solutions to $x + 2 < 9$ are the numbers less than 7 (and only those numbers).

1. For each of these inequalities, perform operations to get equivalent inequalities until you obtain one that shows the solution.

   a. $2x + 5 < 8$

   b. $3x - 2 \geq x + 1$

   c. $3x + 7 \leq 5x - 9$

   d. $4 - 2x > 7 + x$

Part II: Two or More Variables

When an inequality has more than one variable, you can't put it into a form that directly describes the solution. But you can often write the inequality in a simpler equivalent form, such as by combining terms.

For example, suppose you start with the inequality

$$9x - 4y - 2 \geq 3x + 10y + 6$$

You can do these steps to get a sequence of simpler equivalent inequalities.

$9x - 2 \geq 3x + 14y + 6$	(adding $4y$ to both sides)
$6x - 2 \geq 14y + 6$	(subtracting $3x$ from both sides)
$6x \geq 14y + 8$	(adding 2 to both sides)

Because all the coefficients in the inequality $6x \geq 14y + 8$ are even, you can do the additional step of dividing both sides by 2, to get $3x \geq 7y + 4$. Each of the inequalities in the sequence is equivalent to the original inequality, but $3x \geq 7y + 4$ seems to be the simplest of them all.

2. a. Find numbers for x and y that fit the inequality $3x \geq 7y + 4$.

   b. Substitute the numbers that you found in Question 2a into the original inequality, $9x - 4y - 2 \geq 3x + 10y + 6$, and verify that they make it true.

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c. Find numbers for x and y that do not fit the inequality $3x \geq 7y + 4$.

d. Substitute the numbers that you found in Question 2c into the original inequality, $9x - 4y - 2 \geq 3x + 10y + 6$, and verify that they make it false.

e. Explain why steps a through d are not enough to prove that the two inequalities are equivalent.

3. For each of the next three inequalities, perform appropriate operations to get simpler equivalent inequalities.

a. $x + 2y > 3x + y + 2$

b. $\frac{x}{2} - y \leq 3x + 1$

c. $0.2y + 1.4x < 10$