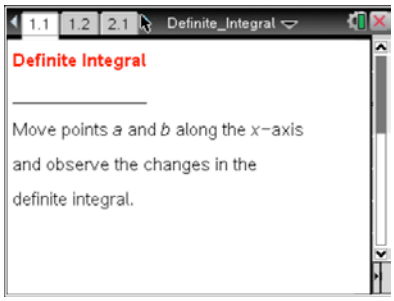
Definite Integral
Student Activity

Name _____

Class _____

Open the TI-Nspire document *Definite_Integral*.



In this activity, you will use a graphical representation to explore the definite integral of a continuous function. You will change the upper and lower limits, *a* and *b*, of the integral $\int_a^b f(x)dx$ and observe the resulting changes in the graph and the value of the definite integral.

Move to page 1.2.

Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

1. The graph shown is of the function $y = f(x)$. The definite integral of $f(x)$ from a to b is given by $\int_a^b f(x)\,dx$. For example, $\int_0^2 f(x)\,dx$is the definite integral of $f(x)$ from 0 to 2, or between $x = 0$ and $x = 2$.

   Drag points *a* and *b* along the *x*-axis to determine the values of the following definite integrals, where *f* is the function shown in the graph.

   a. $\int_0^2 f(x)\,dx =$ _____

   b. $\int_{-3}^2 f(x)\,dx =$ _____

   c. $\int_{-3}^{-2} f(x)\,dx =$ _____

2. Drag point *a* to –3 and move point *b* to determine the following:

   a. For what values of *b* is $\int_{-3}^b f(x)\,dx$ positive? What do you observe about the shaded region and the graph of *f* when $\int_{-3}^b f(x)\,dx$ is positive?

   b. For what values of *b* is $\int_{-3}^b f(x)\,dx$ negative? What do you observe about the shaded region and the graph of *f* when $\int_{-3}^b f(x)\,dx$ is negative?

   c. For what values of *b* does $\int_{-3}^b f(x)\,dx = 0$? What do you observe about the shaded region and the graph of *f* when $\int_{-3}^b f(x)\,dx = 0$?

3. For the function *f* pictured on page 1.2, under what conditions of *a* and *b* in [–5, 5] will the definite integral $\int_a^b f(x)\,dx$ be positive? Negative? Zero? Explain your thinking.

Move to page 2.2.

4. The graph on page 2.2 is of a new function *f*(*x*) and the definite integral $\int_a^b f(x)\,dx$. Drag point *a* to –3 (if *a* is not already positioned at –3).
   a. Without dragging point *b*, for what values of *b* do you think $\int_{-3}^b f(x)\,dx$ will be positive? Negative? Zero? Explain your predictions.

   b. Drag point *b* to test your predictions. Describe what you observed in the graph of *f* that confirmed or contradicted your prediction.

5. For the function *f*(*x*) pictured on page 2.2, under what conditions of *a* and *b* in [–5, 5] will the definite integral $\int_a^b f(x)\,dx$ be positive? Negative? Zero? Explain your thinking.

6. Based on your observations on pages 1.2 and 2.2, for any continuous function *f* on an interval [*c*, *d*] and for *a* and *b* in [*c*, *d*], when will the definite integral $\int_a^b f(x)\,dx$ be positive? Negative? Zero? Clearly explain your generalization.

7. The definite integral $\int_a^b f(x)\,dx$ is often described as "the area under the curve $y = f(x)$ between $x = a$ and $x = b$." What problems do you see with this definition?